# Programming quantum computers - Quantum teleportation

# **Initial settings**

Loading necessary libraries:

```
In [1]: import qiskit
from qiskit.visualization import plot_bloch_multivector, plot_histogram, plot_circuit_layout
from math import sin, cos, pi, sqrt
from random import random, seed
import numpy as np
from qiskit import IBMQ
from qiskit.providers.aer import AerSimulator
import matplotlib.pyplot as plt
from qiskit.transpiler.passes import RemoveBarriers
```

To be able to access IBM quantum devices, one needs to initialize it. For the first time one needs to use save\_account() command with your IBMQ token that can be obtained on the IBMQ page. Later on using load\_account() is used for the initialization.

```
In [2]: # IBMQ.save_account("TOKEN")
IBMQ.load_account()
```

Out[2]: <AccountProvider for IBMQ(hub='ibm-q', group='open', project='main')>

Here are some global definitions that we will use.

```
IBM DEVICE = "ibmq manila"
In [3]:
        SHOTS = 5000
        USE_REPO = True # If repository is used, it will use JOB_IDS to download data
                         # This works only for the user that performed the computations
                         # When using the file with new account, use the False flag
                          # that will rewrite the JOB IDS
        JOB IDS = { 'standard': '62b9b84540f1545c84afc1c7',
                    'no_cif': '62b9b85e0133394a64dde31f',
                    'no_int_measure': '62b9b8776a05a5425c8dca5b',
                    'bad mapping': '62b9b8c55a1cb03067774e4b',
                    'best': '62b9b8e8b343f0f51c5f1326'}
        # Device to be simulated
        DEVICE = IBMQ.get_provider().get_backend(IBM_DEVICE)
        # Extracted simulator for the device
        DEVICE_SIM = AerSimulator.from_backend(DEVICE)
        # Perfect simulator
        IDEAL_SIM = qiskit.Aer.get_backend('qasm_simulator')
```

## Bits and pieces we will use

Our tasks will be very specific - we will design our experiments so that the preferred outcome will be always 0. To this end we design all our helper functions.

### **Experiments classes**

The following class just simplifies manipulation of experiments. How to perform an experiment is also shown later.

```
class Experiment:
In [4]:
            Experiment manipulation class.
            Attributes:
             - name: name of the experiment
              - circuit: circuit to be executed
             - shots: number of shots in the execution
             - ideal: NumPy array of success probabilities (measuring 0) using ideal simulator
             - sim: NumPy array of success probabilities (measuring 0) using device simulator [genera
             - result: NumPy array of success probabilities (measuring 0) [or None]
             - job_id: IBMQ job ID for data retrieval [or None]
            Methods:
             - get_device_results: (i) if job_id is None, experiment is submitted to the device, else
                                    (ii) if results are None, job is retrieved from the device
              - status: prints status of the experiment
            def __init__(self, name, circuit, shots=SHOTS):
                self.name = name
                self.circuit = circuit
                self.shots = shots
                # job can be submitted with (usually up to 100) circuits [depends on the device]
                job = qiskit.execute([self.circuit] * 100, DEVICE_SIM, shots=self.shots)
                self.sim = Experiment._extract_bobs_success(job.result().get_counts())
                job = qiskit.execute([self.circuit] * 100, IDEAL_SIM, shots=self.shots)
                self.ideal = Experiment._extract_bobs_success(job.result().get_counts())
                self.result = None
                self.job id = JOB IDS.get(self.name, None) if USE REPO else None
            # Extracts success rate (of measuring 0 on Bob's qubit)
            @staticmethod
            def _extract_bobs_success(counts_lst, pos=0):
                succ = []
                for counts in counts 1st:
                    bob_counts = {"0":0, "1":0}
                    for key in counts:
                         bob_key = key[pos]
                         bob_counts[bob_key] += counts[key]
                    succ.append(bob_counts["0"] / (bob_counts["0"] + bob_counts["1"]))
                return np.array(succ)
            # (i) if job_id is None, experiment is submitted to the device, else
            # (ii) if results are None, job is retrieved from the device
            def get device results(self):
                if self.job id is None:
                    job = qiskit.execute([self.circuit] * 100, DEVICE, shots=SHOTS)
                    self.job_id = job.job_id()
                    JOB IDS[self.name] = self.job id
                    print(f"Job submitted to {IBM_DEVICE} under ID {job.job_id()}")
                    return
                if self.result is None:
                    job = DEVICE.retrieve_job(self.job_id)
                    self.result = Experiment._extract_bobs_success(job.result().get_counts())
                    print(f"Precomputed job successfully downloaded from repository")
            # Prints status of the experiment
            def status(self, old=True):
                print(f"EXPERIMENT: {self.name}")
                print(f"Ideal success rate: {100 * self.ideal.mean():.2f} ± {100 * self.ideal.std():.
```

```
print(f"Simulated device: {100 * self.sim.mean():.2f} ± {100 * self.sim.std():.2f} %"
                 if self.result is not None:
                     print(f"Actual IBMQ device ({IBM DEVICE}): {100 * self.result.mean():.2f} ± {100
                 else:
                     print(f"IBMQ runs not available.")
                 depth = self.circuit.depth()
                 ops = self.circuit.count_ops()
                 double = ops.get("cx", 0) + ops.get("cz", 0)
                 meas = ops["measure"]
                 single = sum(ops[x] for x in ops if x not in ["barrier", "measure", "cx", "cz"])
                 print(f"The circuit has depth {depth}, contains {single} one-qubit operations, {doubl
In [5]:
        class Experiments:
             .....
            Class for the collection of experiments
            Attributes:
             - experiments: dictionary of experiments [name:experiment]
            Methods:
             - add: adds experiment if not in experiments
             - remove: removes experiment from experiments by name
             - get: returns experiment by name
               status: prints statuses of all experiments
             .....
            def __init__(self):
                 self.experiments = {}
            def add(self, exp):
                 assert exp.name not in self.experiments.keys()
                 self.experiments[exp.name] = exp
             def remove(self, name):
                 if name in self.experiments.keys():
                     del self.experiments[name]
            def get(self, name):
                 return self.experiments.get(name, None)
            def status(self):
                 for e in self.experiments.values():
                     e.status()
                     print()
```

### Random state preparator

We want to prepare a function that will return a Qiskit gate for random state preparation. We shall use  $U_3$  unitary with randomly chosen angles  $\theta$ ,  $\phi$  and  $\lambda$ . As we intend to test the correctness of the teleportation, the function will return both U and  $U^{\dagger}$ .

```
In [6]: def random_gate(s=None):
    seed(s)

    # Choose the angles randomly
    theta = 2*pi*random()
    phi = pi*random()
    lam = pi*random()

    # Composing the gate U
    c = qiskit.QuantumCircuit(1)
    c.u(theta, phi, lam, 0)

    # Composing the gate U^\\dagger
    c_dag = qiskit.QuantumCircuit(1)
    c_dag.u(-theta, -lam, -phi, 0)
```

```
seed()
return c.to_gate(label="U"), c_dag.to_gate(label="U+")
```

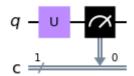
## Simple experiments to see how Qiskit works

The circuit runs on a qubit register initialized to state  $|0\rangle$ . Then we perform the computation by using various quantum gates (similarly as in the classical computation). To get the results we perform measurements on chosen qubits. Measurements are in general stochastic producing in each run either a 0 or a 1 for each qubit.

```
In [7]: u, u_dag = random_gate()
```

```
# Define the circuit
c = qiskit.QuantumCircuit(1, 1)
c.append(u, [0])
# Append measurements
c.measure(0, 0)
c.draw(output="mpl")
```

#### Out[7]:



Circuits are submitted as jobs to given backends (devices) and these are run several times to get the statistics. One can submit also a list of circuits.

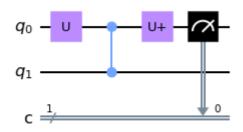
```
In [8]: job = qiskit.execute(c, IDEAL_SIM, shots=SHOTS)
counts = job.result().get_counts()
print(counts)
```

```
{'0': 3478, '1': 1522}
```

Let us see what happens in a slightly more involved case. We add one CZ gate and use  $U^{\dagger}$  to return the first qubit to its original state - if we have somewhere state  $|\psi\rangle$  prepared by U, then applying  $U^{\dagger}$  on it shall return it to state  $|0\rangle$  and so we can measure the success in this case by the probability of getting outcome 0.

```
In [9]: # Define the circuit
c = qiskit.QuantumCircuit(2, 1)
c.append(u, [0])
c.cz(0, 1)
c.append(u_dag, [0])
# Append measurements
c.measure(0, 0)
c.draw(output="mpl")
```

Out[9]:

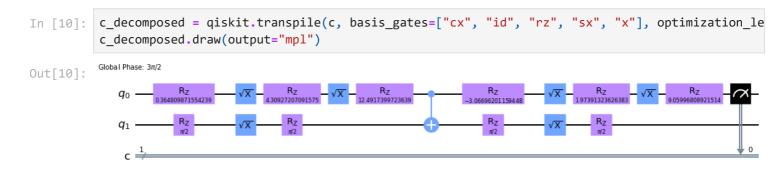


Each specific device uses its own gates set that it can perform. And so if we submit the circuit for the computation, it undergoes many steps before it is fed to the quantum device:

- routing: fitting the circuit to chosen device
- decomposition: replacing gates with the native set
- optimization: reducing the complexity of fitted circuit

These steps are collectively called *transpilation*, but the process is more complex than that c.f. this link.

One of the most important ways of estimating complexity of the circuit is to look at Let's see what happens to the circuit when we decompose it.



Running the circuit on ideal simulator gives following results:

```
In [11]: job = qiskit.execute(c_decomposed, IDEAL_SIM, shots=SHOTS)
counts = job.result().get_counts()
print(counts)
```

```
{'0': 5000}
```

We can make use of the Experiment class where we will automatically extract success rates not only for the ideal simulator, but also for simulated device (with errors) and possibly with submission to actual device.

```
In [12]: e = Experiment("test", c_decomposed)
```

```
In [13]: e.status()
```

```
EXPERIMENT: test
Ideal success rate: 100.00 ± 0.00 %
Simulated device: 97.87 ± 0.21 %
IBMQ runs not available.
The circuit has depth 12, contains 16 one-qubit operations, 1 two-qubit operations and 1 meas
urements.
```

## Teleportation

Teleportation is one of the basic quantum tasks where a state is "teleported" from one qubit to another just by using entanglement and classical communication. Before immersing in the implementation, let us

initialize the Experiments class and the seed for random number generation in the random\_gate, so that the results will be consistent.

```
In [14]: rnd_seed = 42
```

```
In [15]: exp = Experiments()
```

## Standard circuit

Standard teleportation circuit consists of Alice with two qubits and Bob having single qubit. Before Alice wants to send her state to Bob, the two of them make a maximally entangled Bell state between one of Alice's qubits and Bob's qubit. Alice and Bob are then free to go as far as they want. When Alice wants to send her qubit, she performs a specific (Bell) measurement on her two qubits - one with the unknown state and the other one, which shares entanglement with Bob's qubit. Alice will get two bits of information that she sends to Bob. Bob, after receiving the two bits (which are completely random) adjusts his state and in the end he is supposed to get the state that Alice was sending. Alice's qubits are in a random state of the measurement basis.

The circuit thus consists of four parts:

- 1. Creating entanglement
- 2. Preparation of the teleported state
- 3. Bell measurement on Alice's qubits
- 4. Correction of Bob's state based on Alice's result

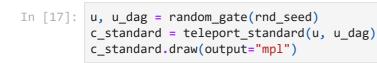
We will implement also additional step that we discussed above - as expect the final Bob's state to be prepared by unitary U, we will "undo" it by  $U^{\dagger}$  and the measurement should be in state 0. And so the fifth part of the circuit is:

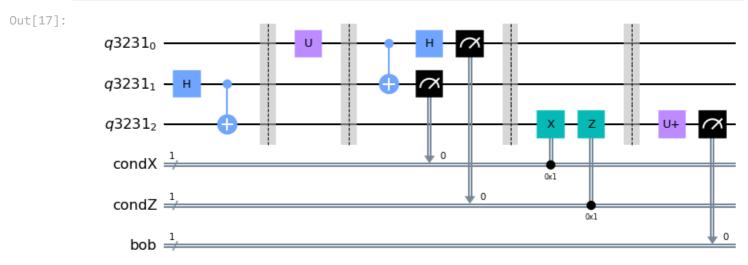
1. Rotating the state to the measurement basis and measure

```
In [16]:
         def teleport standard(u, u dag):
             qreg = qiskit.QuantumRegister(3)
             # We will use three separate classical registers (bits)
             # for the two bits of information Alice sends to Bob
             # and the third for potential measurement of Bob
             cregx = qiskit.ClassicalRegister(1, name="condX")
             cregz = qiskit.ClassicalRegister(1, name="condZ")
             cregbob = qiskit.ClassicalRegister(1, name="bob")
             teleport = qiskit.QuantumCircuit(qreg, cregx, cregz, cregbob)
             # Step 1: Creating entanglement
             teleport.h(1)
             teleport.cx(1, 2)
             teleport.barrier()
             # Step 2: Preparation of the telported state
             # (random state will be passed in the argument of the function)
             teleport.append(u, [0])
             teleport.barrier()
             # Step 3: Bell measurement
             teleport.cx(0, 1)
             teleport.h(0)
             teleport.measure(0, cregz[0])
             teleport.measure(1, cregx[0])
             teleport.barrier()
```

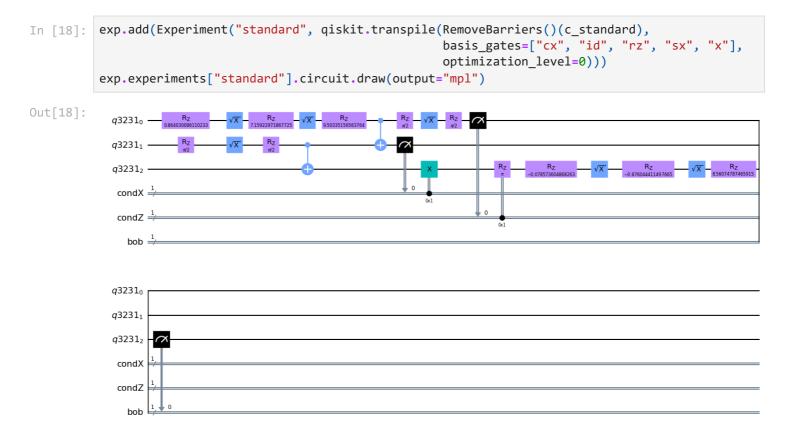
```
# Step 4: Correcting state on Bob;s side
teleport.x(2).c_if(cregx, 1)
teleport.z(2).c_if(cregz, 1)
teleport.barrier()
# Step 5: Bob's measurement
# (If everything is correct, only 0s are expected)
teleport.append(u_dag, [2])
teleport.measure(2, cregbob[0])
return teleport
```

Let's draw the full teleportation circuit with some random state for Alice to send.





Barriers in the picture have both visual function, but also practical. Therefore, we will remove them before using the circuit. We will also decompose the circuit to the native base set of IBM quantum devices and for now we will leave optimization and routing to the standard setting.



Let us see what the quantum computation on real device gives:

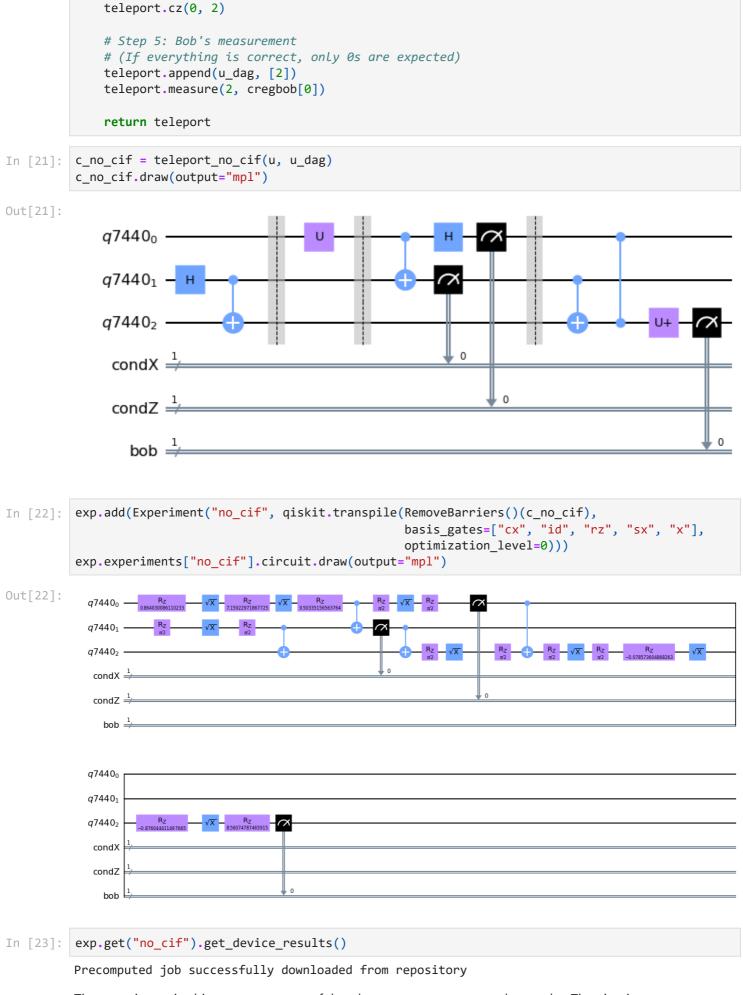
```
In [19]: exp.get("standard").get_device_results()
```

```
_____
IBMOJobFailureError
                                       Traceback (most recent call last)
~\AppData\Local\Temp\1/ipykernel_19744/893574494.py in <module>
----> 1 exp.get("standard").get_device_results()
~\AppData\Local\Temp\1/ipykernel_19744/1700584260.py in get_device_results(self)
           if self.result is None:
    56
    57
                  job = DEVICE.retrieve_job(self.job_id)
---> 58
                  self.result = Experiment._extract_bobs_success(job.result().get_counts())
    59
                  print(f"Precomputed job successfully downloaded from repository")
    60
~\Miniconda3\envs\qiskit\lib\site-packages\qiskit\providers\ibmq\job\ibmqjob.py in result(sel
f, timeout, wait, partial, refresh)
   288
                      else:
                          error_message = ": " + error_message
   289
--> 290
                      raise IBMQJobFailureError(
   291
                          'Unable to retrieve result for job {}. Job has failed{}'.format(
   292
                              self.job_id(), error_message))
IBMQJobFailureError: 'Unable to retrieve result for job 62b9b84540f1545c84afc1c7. Job has fai
led: Instruction bfunc is not supported. Error code: 7001.'
```

Running this circuit on IBM quantum device fails (situation on 27/06/2022) as it does not support classical feedback. It has been just few months since IBMQ started supporting intermediate measurements on some of the devices. So now we can use the fact that the quantum bit that was measured will be in the same (classical) state as was the classical outcome of the measurement; we can do the measurements, but will use the corresponding qubit as the control.

### Removing the classical control

```
In [20]:
         def teleport no cif(u, u dag):
             qreg = qiskit.QuantumRegister(3)
             # We will use three separate classical registers (bits)
             # for the two bits of information Alice sends to Bob
             # and the third for potential measurement of Bob
             cregx = qiskit.ClassicalRegister(1, name="condX")
             cregz = qiskit.ClassicalRegister(1, name="condZ")
             cregbob = qiskit.ClassicalRegister(1, name="bob")
             teleport = qiskit.QuantumCircuit(qreg, cregx, cregz, cregbob)
             # Step 1: Creating entanglement
             teleport.h(1)
             teleport.cx(1, 2)
             teleport.barrier()
             # Step 2: Preparation of the telported state
             # (random state will be passed in the argument of the function)
             teleport.append(u, [0])
             teleport.barrier()
             # Step 3: Bell measurement
             teleport.cx(0, 1)
             teleport.h(0)
             teleport.measure(0, cregz[0])
             teleport.measure(1, cregx[0])
             teleport.barrier()
             # Step 4: Correcting state on Bob; s side
             teleport.cx(1, 2)
```



The experiment in this case was successful and we can now compare the results. The circuit implementable on QPUs is now longer and is suspected to give worse results than the standard circuit would. We see that the estimate is very optimistic, as the actual device gives a success rate by 10% worse than the simulation.

```
EXPERIMENT: standard
Ideal success rate: 100.00 ± 0.00 %
Simulated device: 94.62 ± 0.35 %
IBMQ runs not available.
The circuit has depth 17, contains 18 one-qubit operations, 2 two-qubit operations and 3 meas
urements.
EXPERIMENT: no_cif
Ideal success rate: 100.00 ± 0.00 %
Simulated device: 95.36 ± 0.69 %
Actual IBMQ device (ibmq_manila): 65.42 ± 1.32 %
The circuit has depth 21, contains 22 one-qubit operations, 4 two-qubit operations and 3 meas
urements.
```

#### Without intermediate measurements

We can simplify the circuit even further also it no longer corresponds to the strict conditions of the definition above. The classical communication is replaced by quantum feedback; one can show that the measurements are irrelevant.

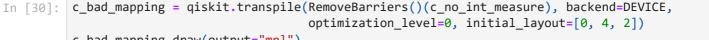
```
In [25]: def teleport no int measure(u, u dag):
             qreg = qiskit.QuantumRegister(3)
             # We will use three separate classical registers (bits)
             # for the two bits of information Alice sends to Bob
             # and the third for potential measurement of Bob
             cregbob = qiskit.ClassicalRegister(1, name="bob")
             teleport = qiskit.QuantumCircuit(qreg, cregbob)
             # Step 1: Creating entanglement
             teleport.h(1)
             teleport.cx(1, 2)
             teleport.barrier()
             # Step 2: Preparation of the telported state
             # (random state will be passed in the argument of the function)
             teleport.append(u, [0])
             teleport.barrier()
             # Step 3: Bell "measurement"
             teleport.cx(0, 1)
             teleport.h(0)
             teleport.barrier()
             # Step 4: Correcting state on Bob;s side
             teleport.cx(1, 2)
             teleport.cz(0, 2)
             # Step 5: Bob's measurement
             # (If everything is correct, only 0s are expected)
             teleport.append(u_dag, [2])
             teleport.measure(2, cregbob[0])
             return teleport
```

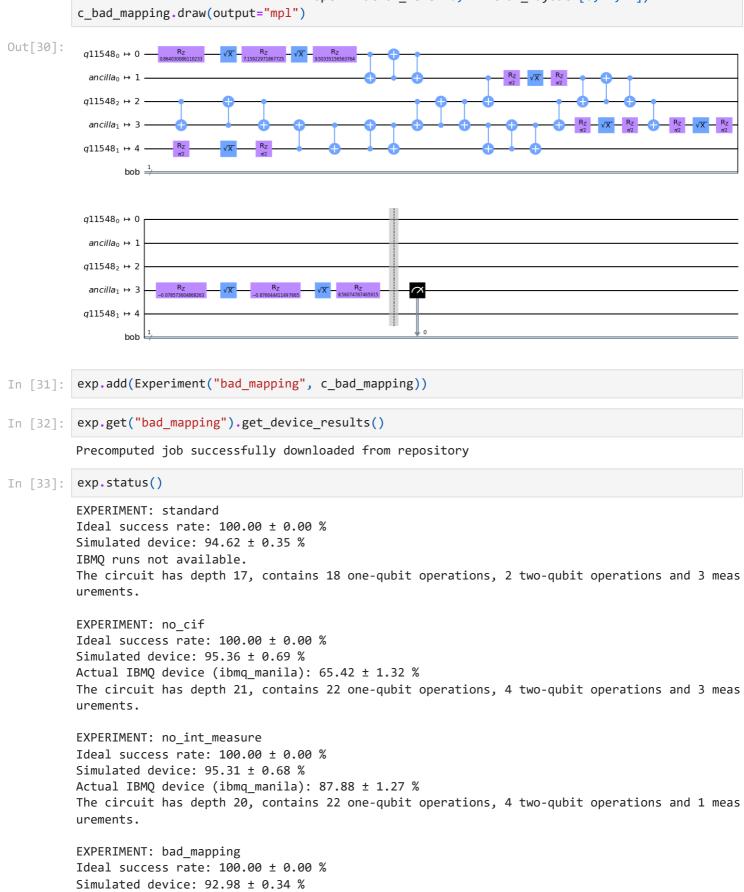




### Including architecture and bad mapping

Part of transpilation is also routing. That is a way of mapping logical qubits to physical qubits. It makes a lot of difference. See for example this mapping: qr[0]-> QB0 , qr[1]-> QB4 , qr[2] -> QB6

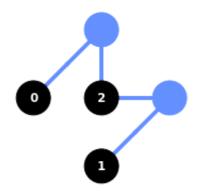




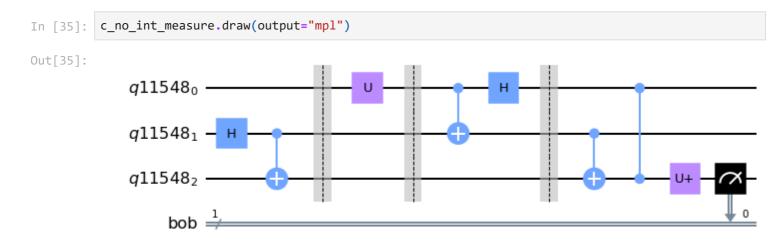
Actual IBMQ device (ibmq\_manila): 86.65 ± 0.45 % The circuit has depth 27, contains 22 one-qubit operations, 22 two-qubit operations and 1 mea surements.

These results are much worse since the routing is very bad. We made use of following qubits:

Out[34]:

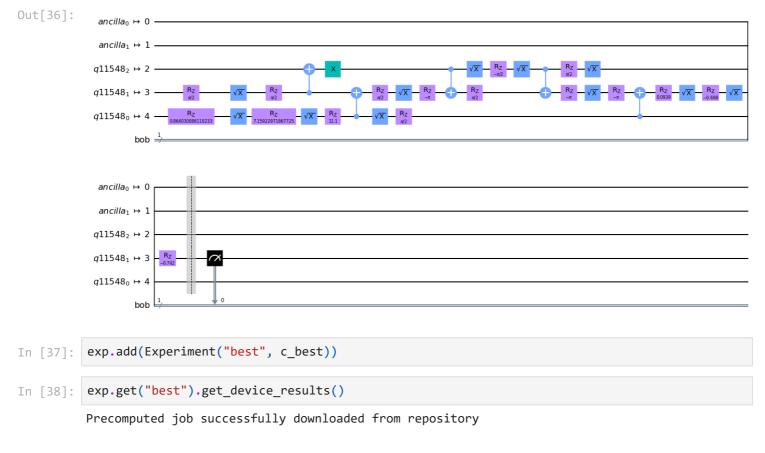


Routing is very important and can make a big difference. Problematic is, however, if the original circuit has CNOT "loops":



## Good mapping and optimization

Finally, after routing and decomposing, one can use various methods to optimize the circuit (single qubit gates might be joined, etc.). Let us have a look at the best automatic optimization in Qiskit (it is probabilistic).



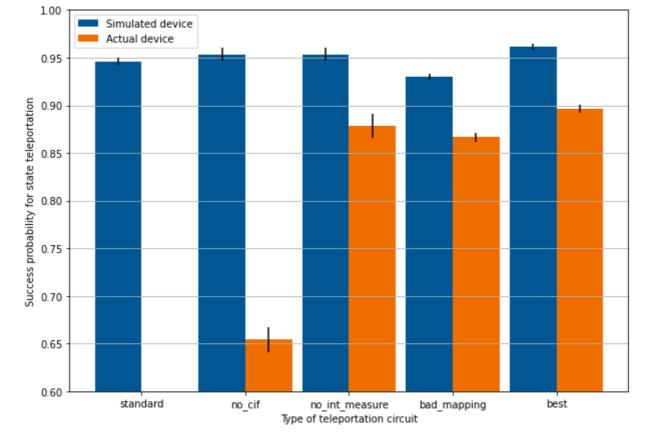
# Wrap-up

Let us have a look at all the data we produced:

In [39]: exp.status()

**EXPERIMENT:** standard Ideal success rate: 100.00 ± 0.00 % Simulated device: 94.62 ± 0.35 % IBMQ runs not available. The circuit has depth 17, contains 18 one-qubit operations, 2 two-qubit operations and 3 meas urements. EXPERIMENT: no\_cif Ideal success rate: 100.00 ± 0.00 % Simulated device: 95.36 ± 0.69 % Actual IBMQ device (ibmg manila): 65.42 ± 1.32 % The circuit has depth 21, contains 22 one-qubit operations, 4 two-qubit operations and 3 meas urements. EXPERIMENT: no int measure Ideal success rate: 100.00 ± 0.00 % Simulated device: 95.31 ± 0.68 % Actual IBMQ device (ibmq\_manila): 87.88 ± 1.27 % The circuit has depth 20, contains 22 one-qubit operations, 4 two-qubit operations and 1 meas urements. EXPERIMENT: bad\_mapping Ideal success rate: 100.00 ± 0.00 % Simulated device: 92.98 ± 0.34 % Actual IBMQ device (ibmq\_manila): 86.65 ± 0.45 % The circuit has depth 27, contains 22 one-qubit operations, 22 two-qubit operations and 1 mea surements. EXPERIMENT: best Ideal success rate: 100.00 ± 0.00 % Simulated device: 96.16 ± 0.28 % Actual IBMQ device (ibmq manila): 89.60 ± 0.42 % The circuit has depth 24, contains 28 one-qubit operations, 5 two-qubit operations and 1 meas urements.

```
In [40]:
         def plot(exp):
             xlabels = list(exp.experiments.keys())
             x = np.array(range(len(xlabels)))
              sim = [exp.get(x).sim.mean() for x in xlabels]
              sim_err = [exp.get(x).sim.std() for x in xlabels]
              res = [exp.get(x).result.mean() if exp.get(x).result is not None else 0 for x in xlabels]
             res_err = [exp.get(x).result.std() if exp.get(x).result is not None else 0 for x in xlabe
             fig, ax = plt.subplots(1, 1, figsize=(10, 7))
             width = 0.45
              ax.bar(x - width / 2, sim, yerr=sim_err,
                     width=width, color="#005794", label="Simulated device")
             ax.bar(x + width / 2, res, yerr=res_err,
                     width=width, color="#f06e00", label="Actual device")
             ax.set_xticks(x, xlabels)
             ax.set ylim((0.6, 1))
             ax.legend()
             ax.yaxis.grid(True)
              ax.set_xlabel("Type of teleportation circuit")
              ax.set_ylabel("Success probability for state teleportation")
              plt.show()
          plot(exp)
```



#### Programing quantum computers workflow

- 1. Identify problem to be solved
- 2. Choose suitable platform and vendor
- 3. Prepare quantum program
- 4. Transpile
- 5. Submit

Points 4.-5. are sometimes called quantum compilation.

#### Transpilation:

- routing: fitting the circuit to chosen device
- decomposition: replacing gates with the native set
- optimization: reducing the complexity of fitted circuit

We could see, that routing and optimization play a crucial role in current devices and probably will play for a long time. Just like GPU computing, we can do computations with pre-configured approaches (which will get better over time), but if we will want to get the most out of the QPU, we will have to consider QPU's architecture, quality and (potentially) speed.

```
In [42]: # If you did your own coalculations, replace the JOB_IDS by the following
# dictionary to have the results accessible
JOB_IDS
Out[42]: {'standard': '62b9b84540f1545c84afc1c7',
    'no_cif': '62b9b85e0133394a64dde31f',
    'no_int_measure': '62b9b8776a05a5425c8dca5b',
    'bad_mapping': '62b9b8c55a1cb03067774e4b',
    'best': '62b9b8e8b343f0f51c5f1326'}
```