Invitation to Quantum Information

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Quantum algorithms and software / VTT

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  - State (esp. one qubit)
  - Evolution
  - Measurement
- Two qubits:
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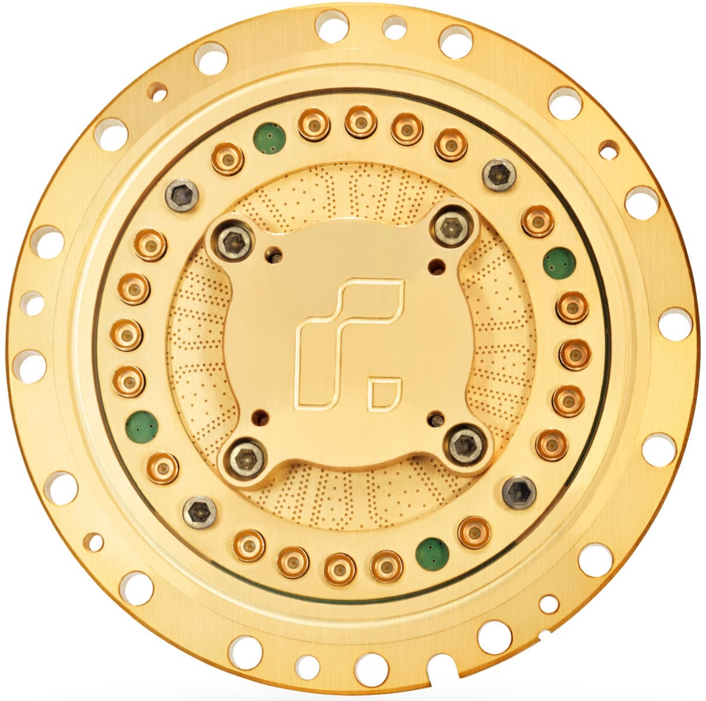
  - EPR paradox and non-locality
- Conclusion (usefulness)



INTRODUCTION

The road to quantum information

Second quantum revolution

- First quantum revolution (20th century): understanding the laws of quantum theory and finding direct uses of observed phenomena

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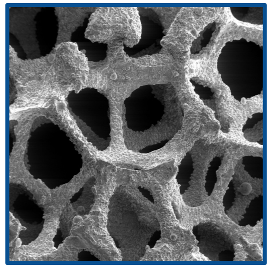
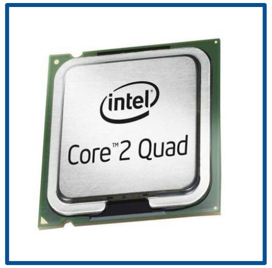
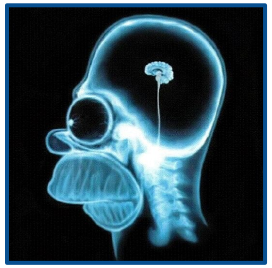
- Second quantum revolution (now): using quantum theory to prepare conditions for manipulation and targeted use of quantum systems

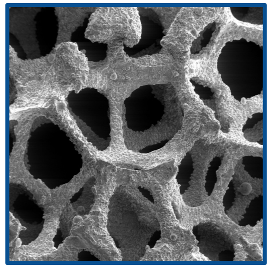
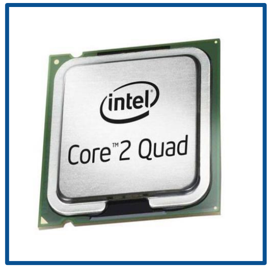

Quantum computation

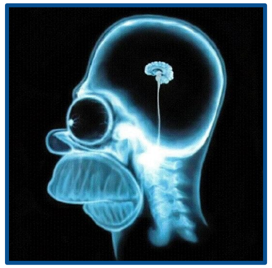
Quantum assisted communication

Quantum sensing

Second quantum revolution

- First quantum revolution (20th century): understanding the laws of quantum theory and finding direct uses of observed phenomena
- Second quantum revolution (now): using quantum theory to prepare conditions for manipulation and targeted use of quantum systems
- New progress requires new language – theory of quantum information


QUANTUM ELEMENTS

Building blocks of understanding quantum

State, evolution and measurement…

State preparation

Evolution

Measurement

Quantum states

- State is an element from Hilbert space $\mathscr{H}$
  1. Vector space over $\mathbb{C}$; vectors are $|\psi\rangle$ (called ket)
  2. Has an inner product $\langle\phi|\psi\rangle$ mapping pairs of vectors to $\mathbb{C}$
     - Positivity: $\langle\psi|\psi\rangle > 0$ for $|\psi\rangle \neq 0$
     - Linearity: $\langle\phi|(a|\psi_1\rangle + b|\psi_2\rangle) = a\langle\phi|\psi_1\rangle + b\langle\phi|\psi_2\rangle$
     - Skew symmetry: $\langle\phi|\psi\rangle = \langle\psi|\phi\rangle^*$
  3. Complete in norm $\|\psi\| = \langle\psi|\psi\rangle^{1/2}$
- Superposition: $|\psi\rangle = a|0\rangle + b|1\rangle + \ldots = \begin{pmatrix} a \\ b \\ \ldots \end{pmatrix}$
- Normalization: $\langle\psi|\psi\rangle = 1 \Rightarrow |a|^2 + |b|^2 + \ldots = 1$

Too complicated for what we need

State is the most complete description we have of the quantum system

$$|\psi\rangle$$

From bit to qubit (two-level q. state)

bit

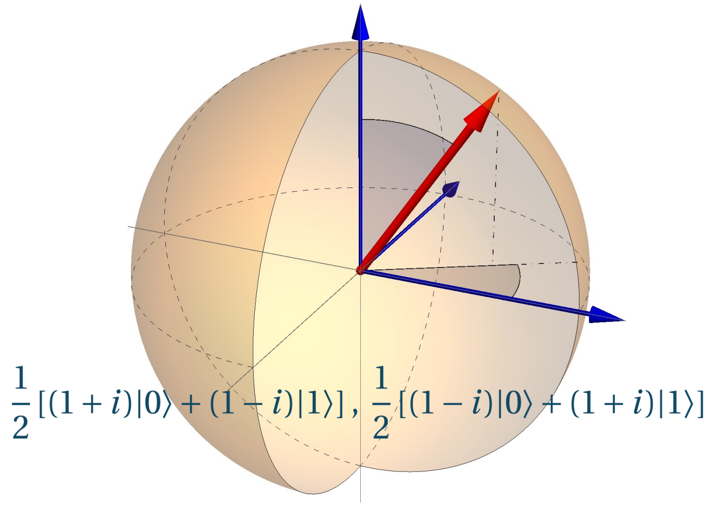
1

0

(p-bit)
probabilistic bit

(qubit)
quantum bit

1



0

Qubit

- State is an element from Hilbert space: $\mathscr{H} = \mathbb{C}^2$
- Orthonormal basis elements: $|0\rangle,\ |1\rangle$
- There are other bases:

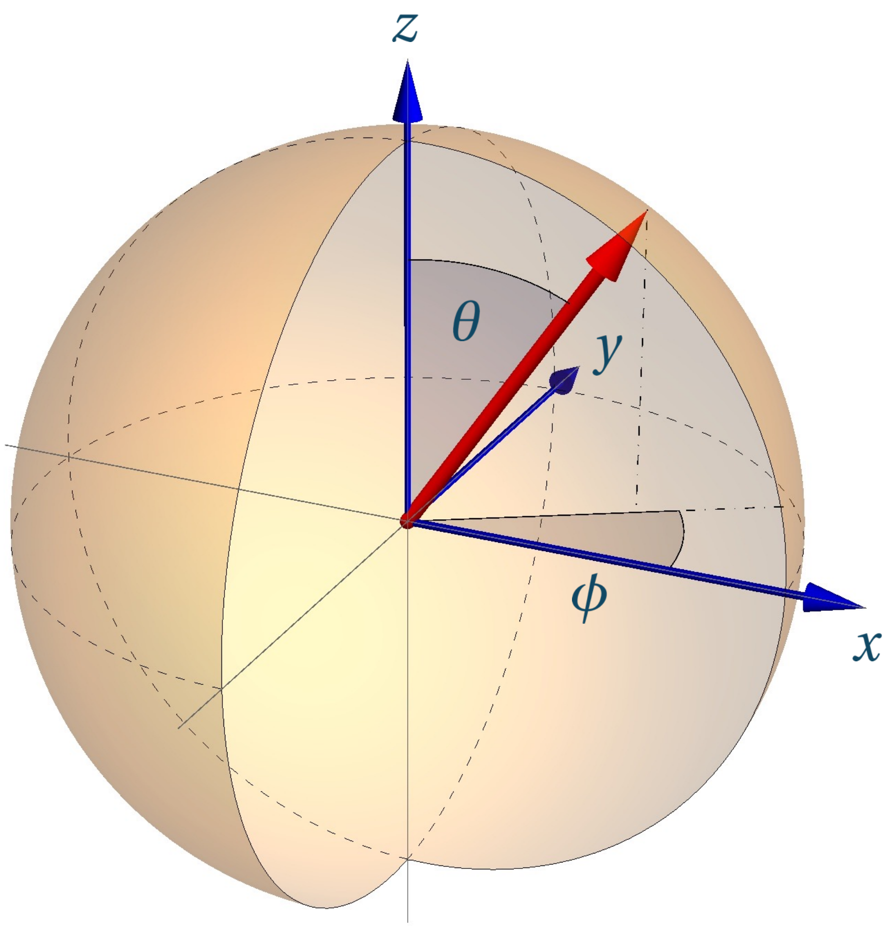
$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),\ |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$\frac{1}{2}\left[(1+i)|0\rangle + (1-i)|1\rangle\right],\ \frac{1}{2}\left[(1-i)|0\rangle + (1+i)|1\rangle\right]$$

- Bloch sphere (up to the global phase):

$$|\psi\rangle = \cos\frac{\theta}{2}e^{-i\frac{\phi}{2}}|0\rangle + \sin\frac{\theta}{2}e^{i\frac{\phi}{2}}|1\rangle$$

- Possible realizations: spin-½ particles, light polarizations, nuclear spins, Josephson junctions, quantum dots, …



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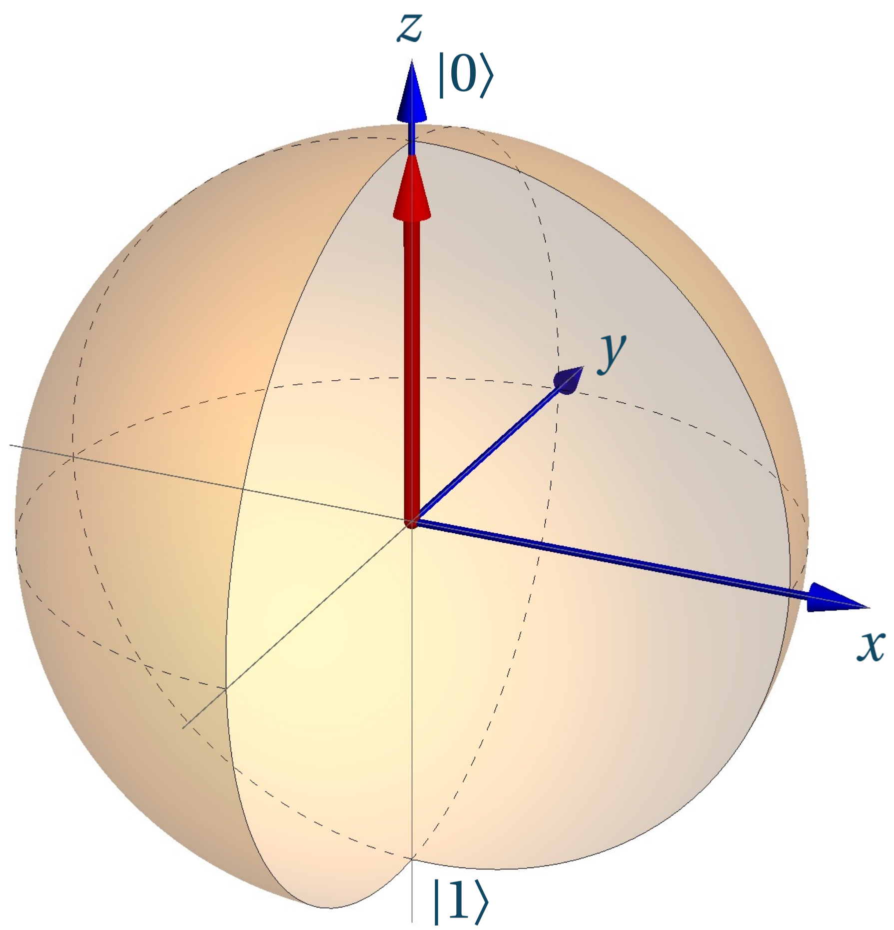

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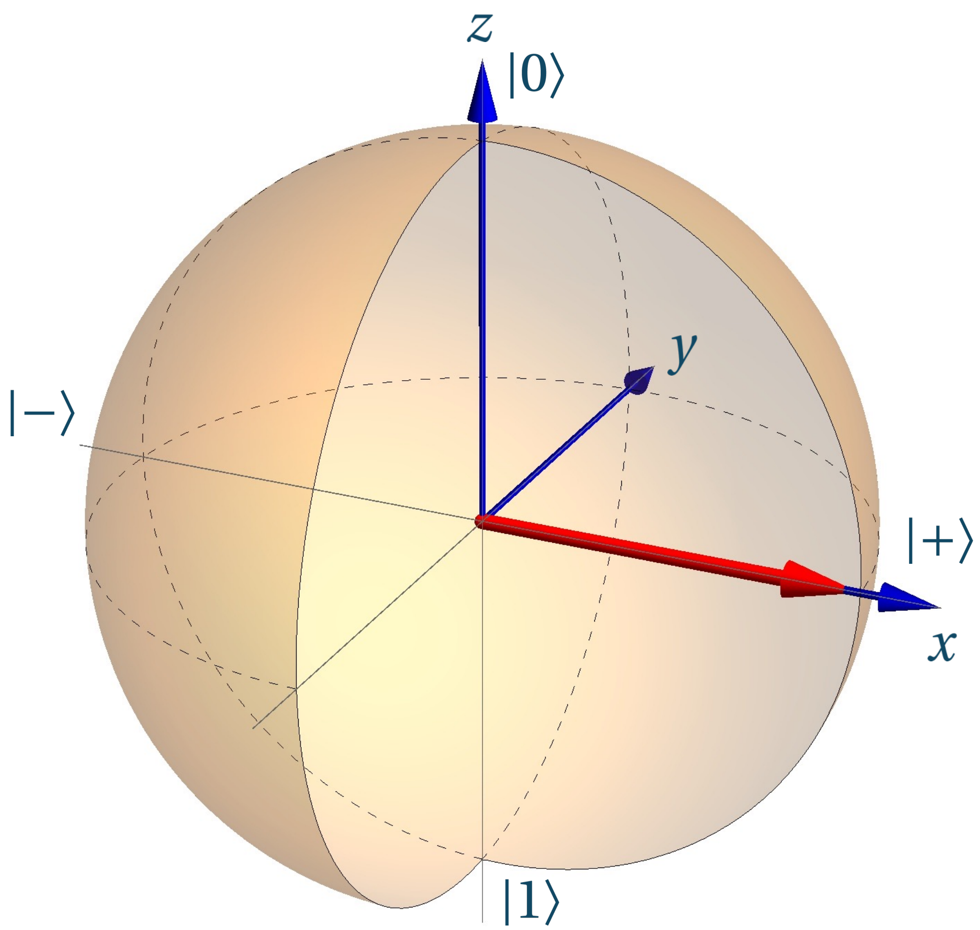

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Different bases

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),\ |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

What we know so far

- State is the most complete description we have of the quantum system
- It is more complex than a classical state

State, evolution and measurement…

State preparation

Evolution

Measurement

State evolutions

- Changes to systems are described as application of some transformation on our state: $U|\psi\rangle$
- Here *U* is a unitary operator (matrix), i.e. $U^{\dagger}U = \mathbb{1}$
- And U is always some Hamiltonian evolution *H* for specific time *t:*

$$U = e^{iHt}$$

- Unitarity conserves normalization and makes computation reversible
- In this lecture we will not talk about decoherence effects or non-unitary evolutions to keep the things simple

Qubit evolutions

- Similarly as qubits, we can express also qubit evolutions in Bloch representation (up to a phase):

$$U(\hat{n},\omega) = \mathrm{e}^{-\mathrm{i}\frac{\omega}{2}\hat{n}\cdot\vec{\sigma}} = \mathbb{1}\cos\frac{\omega}{2} - \mathrm{i}\hat{n}\cdot\vec{\sigma}\sin\frac{\omega}{2}$$

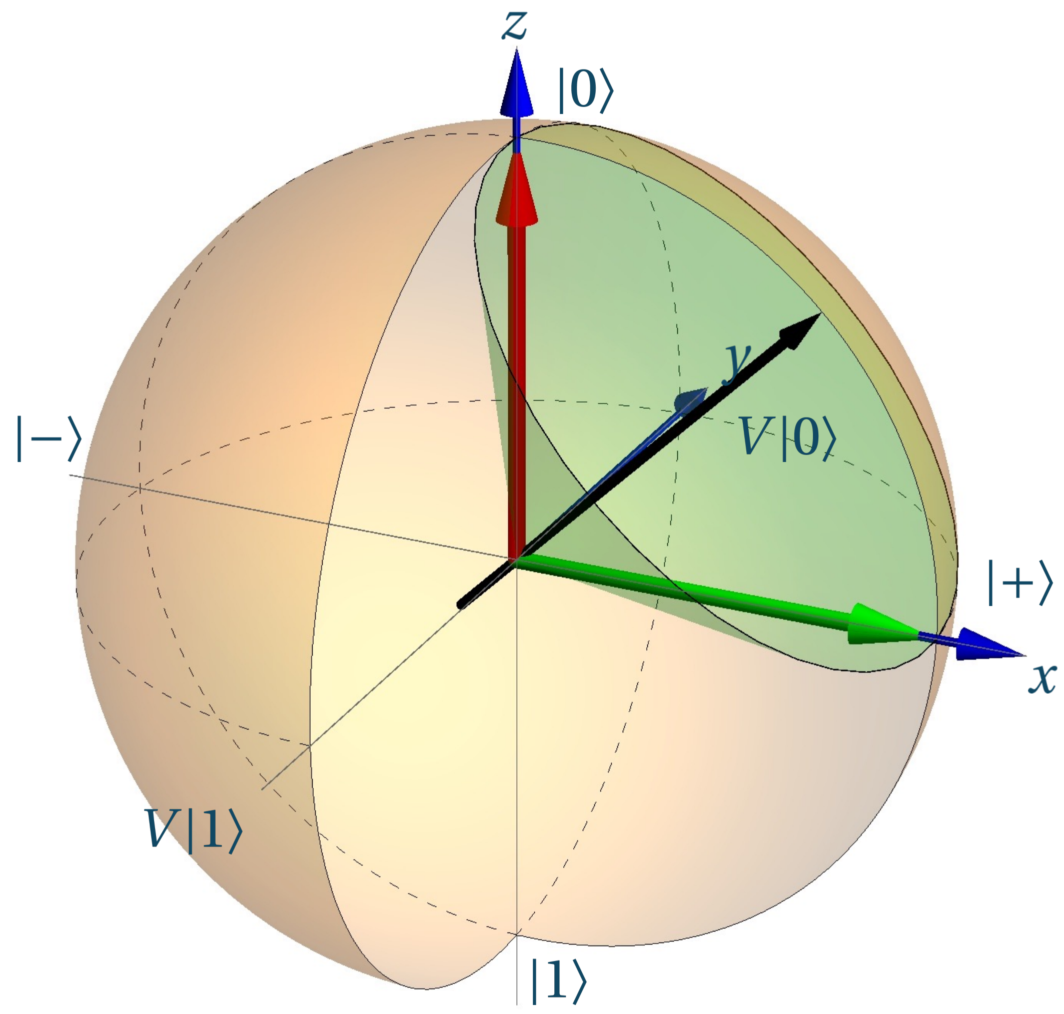
- Here $\hat{n}$ represent a unit vector, ω is the angle and $\vec{\sigma}$ is a vector of Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Rotation around *x*

Rotation around y

Rotation around z



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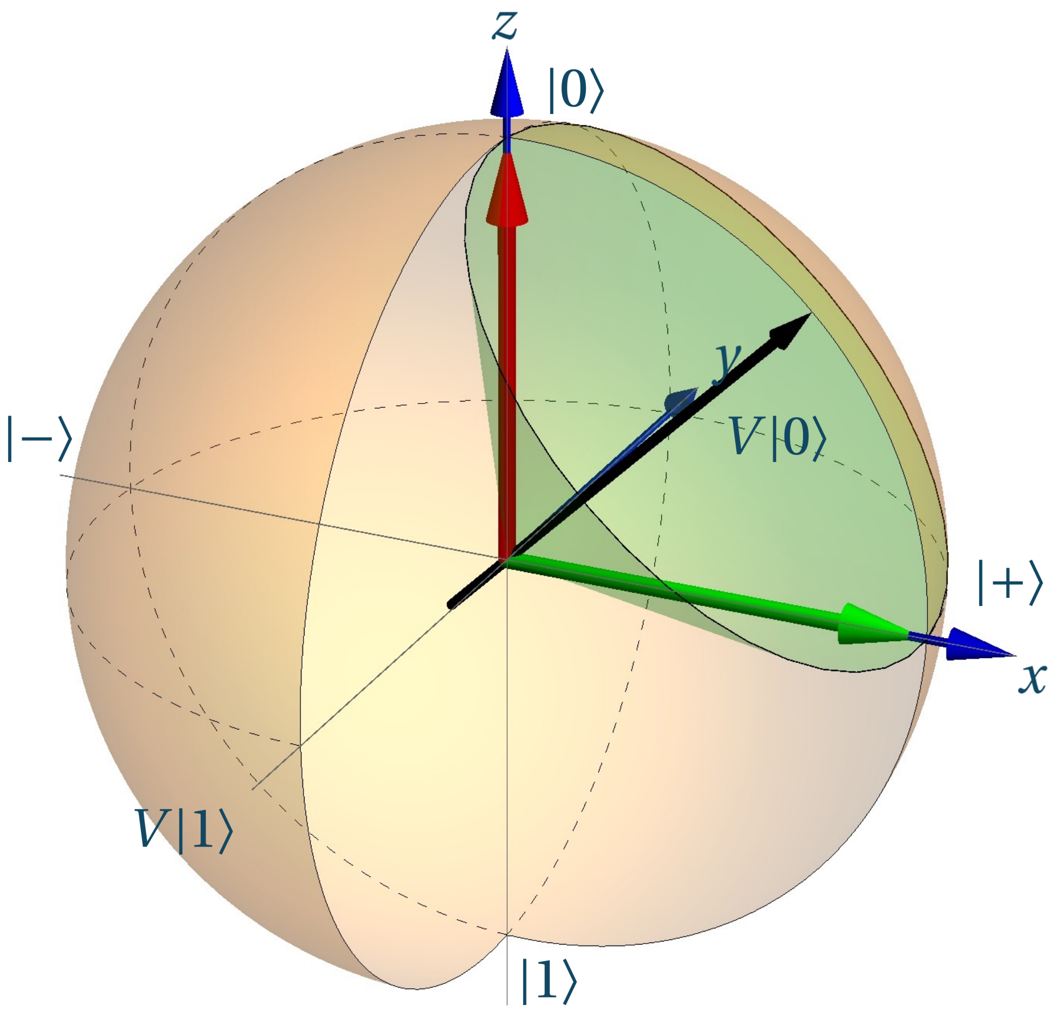
- Hadamard matrix: $H = \sqrt{\mathbb{1}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
- Every unitary U defines a basis

$$\{U|0\rangle, U|1\rangle\}$$



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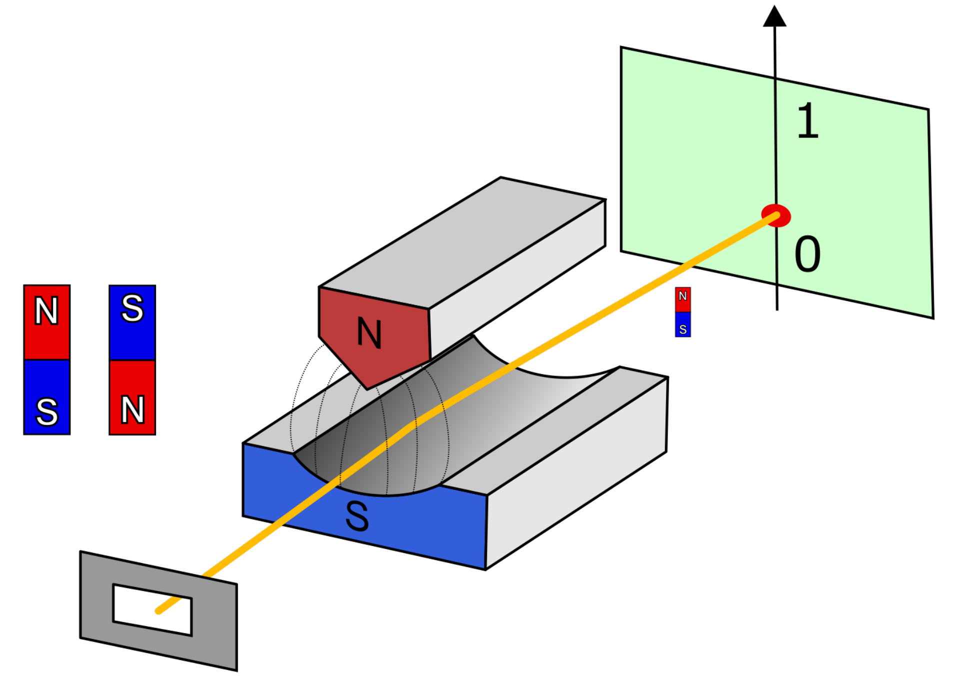
State, evolution and measurement…

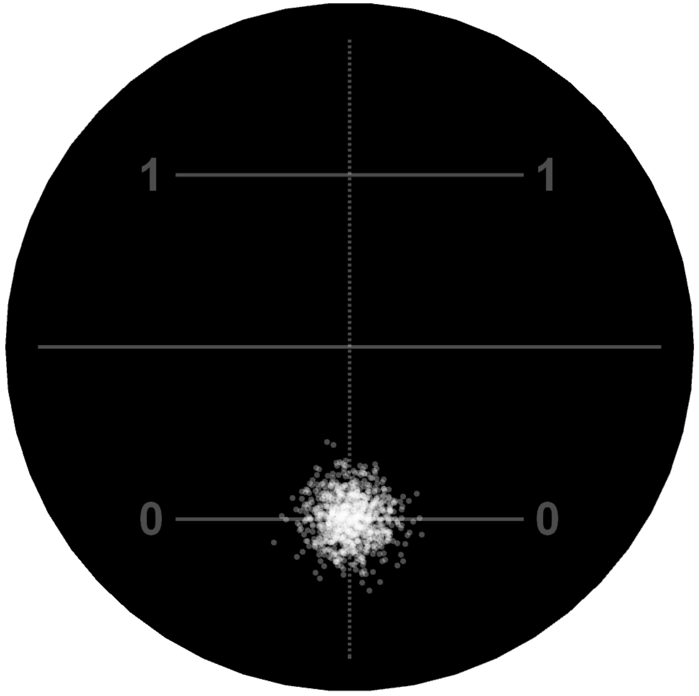
State preparation

Evolution

Measurement

Stern-Gerlach experiment





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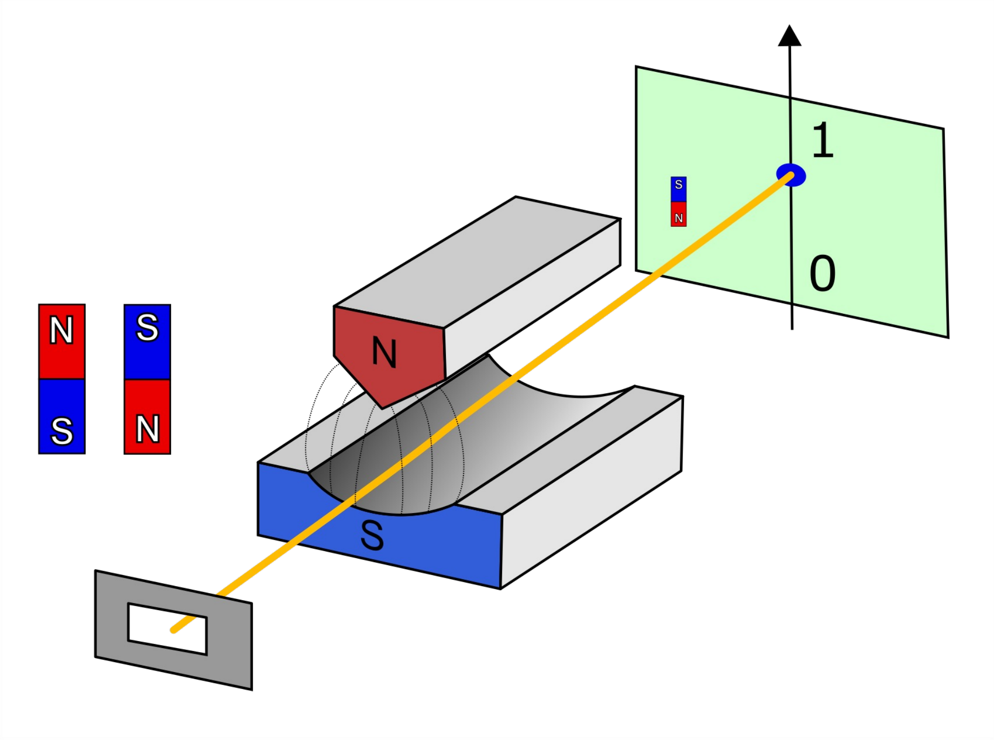

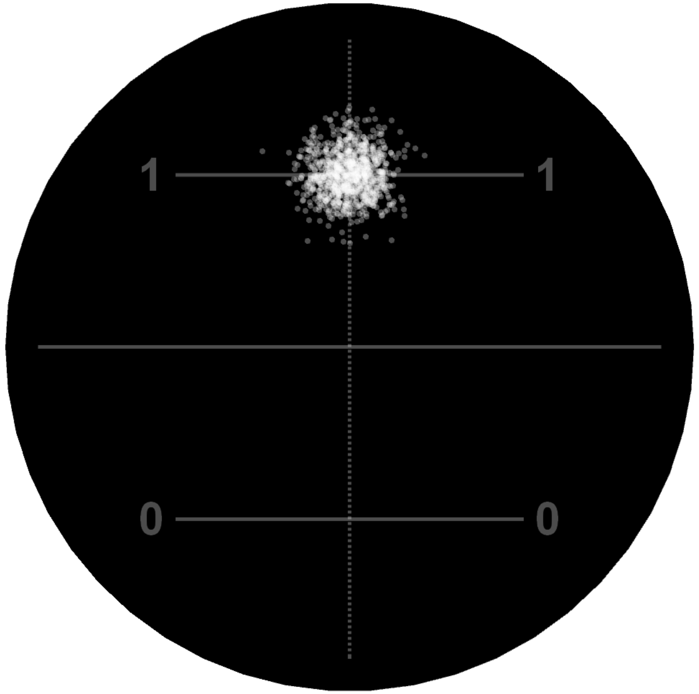

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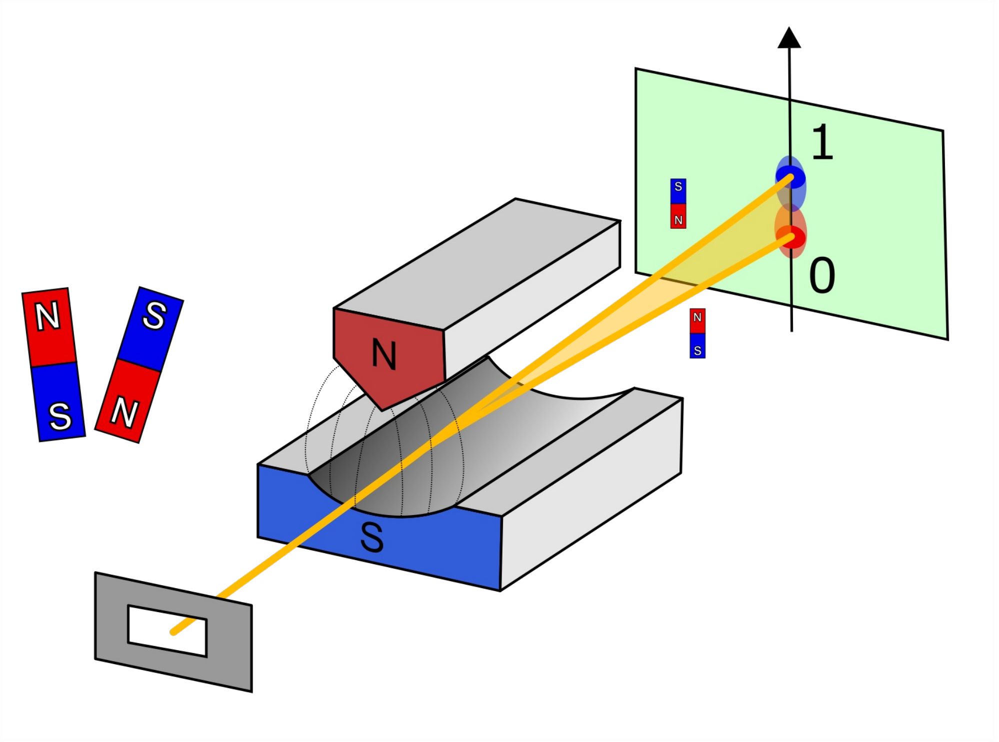

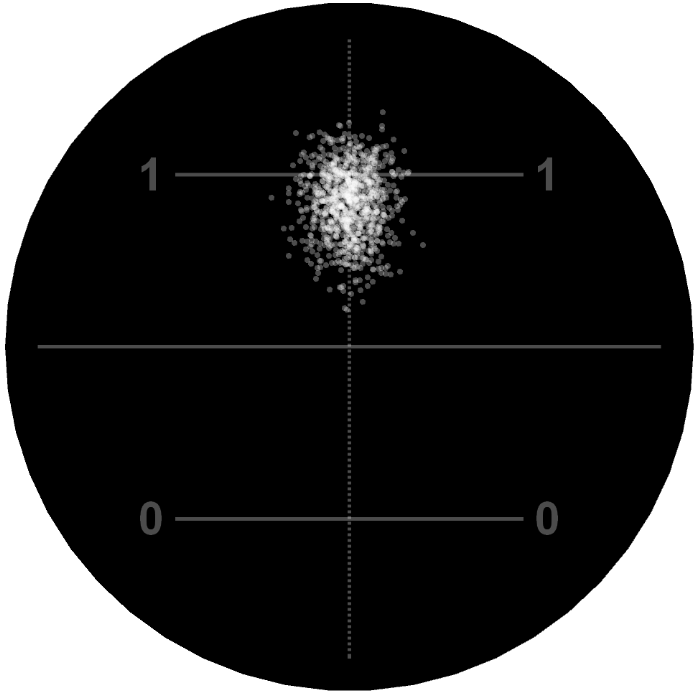

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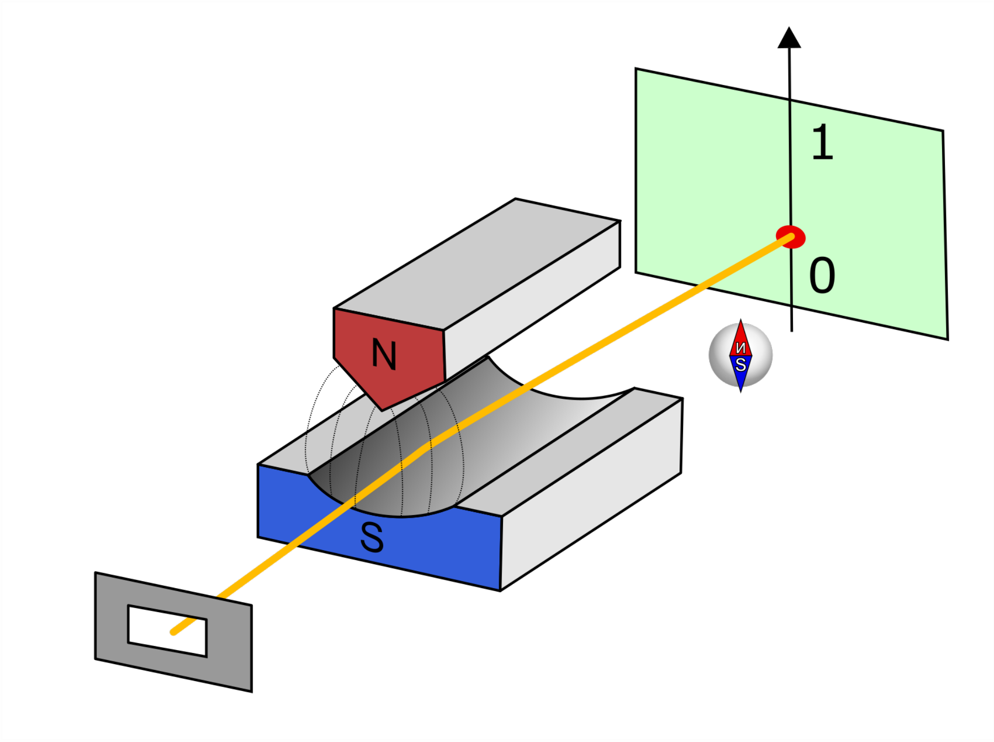

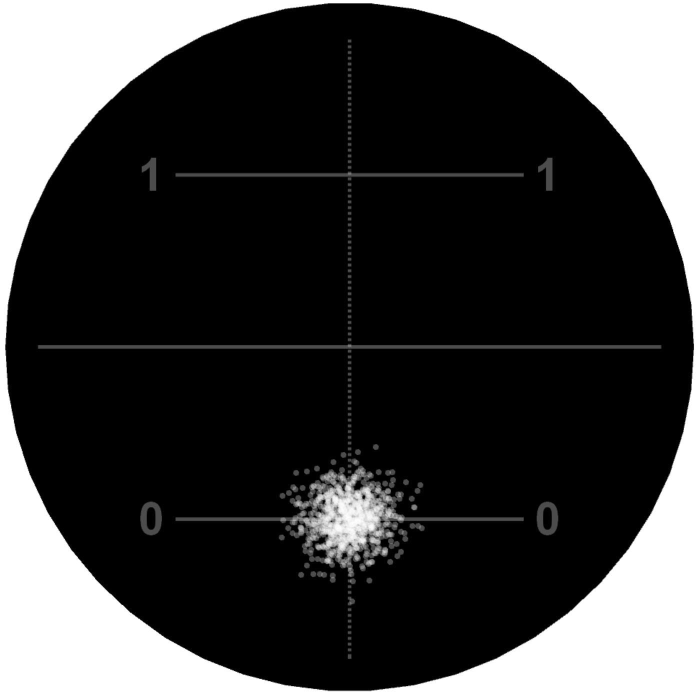

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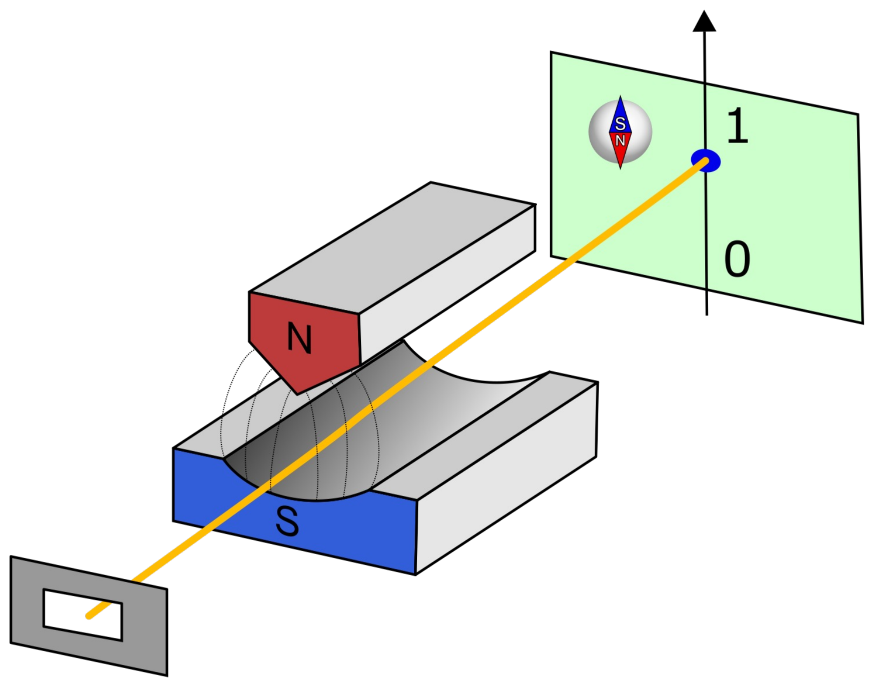

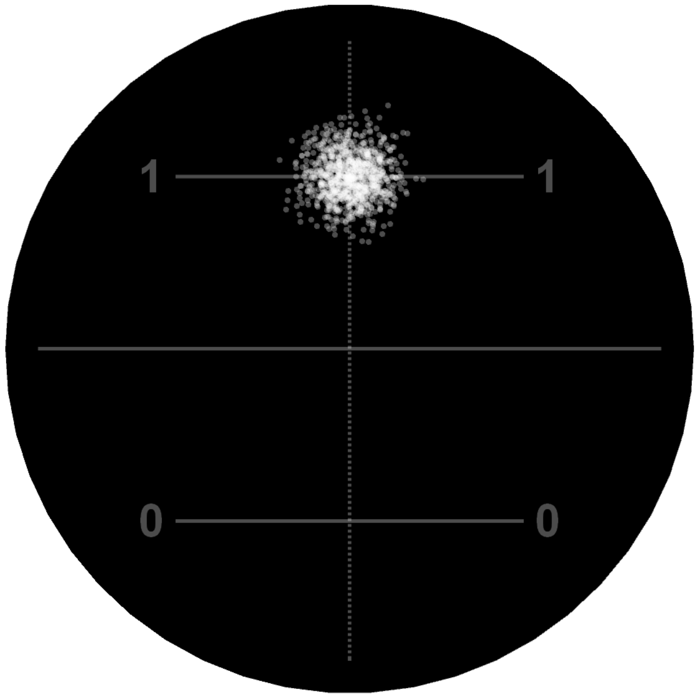

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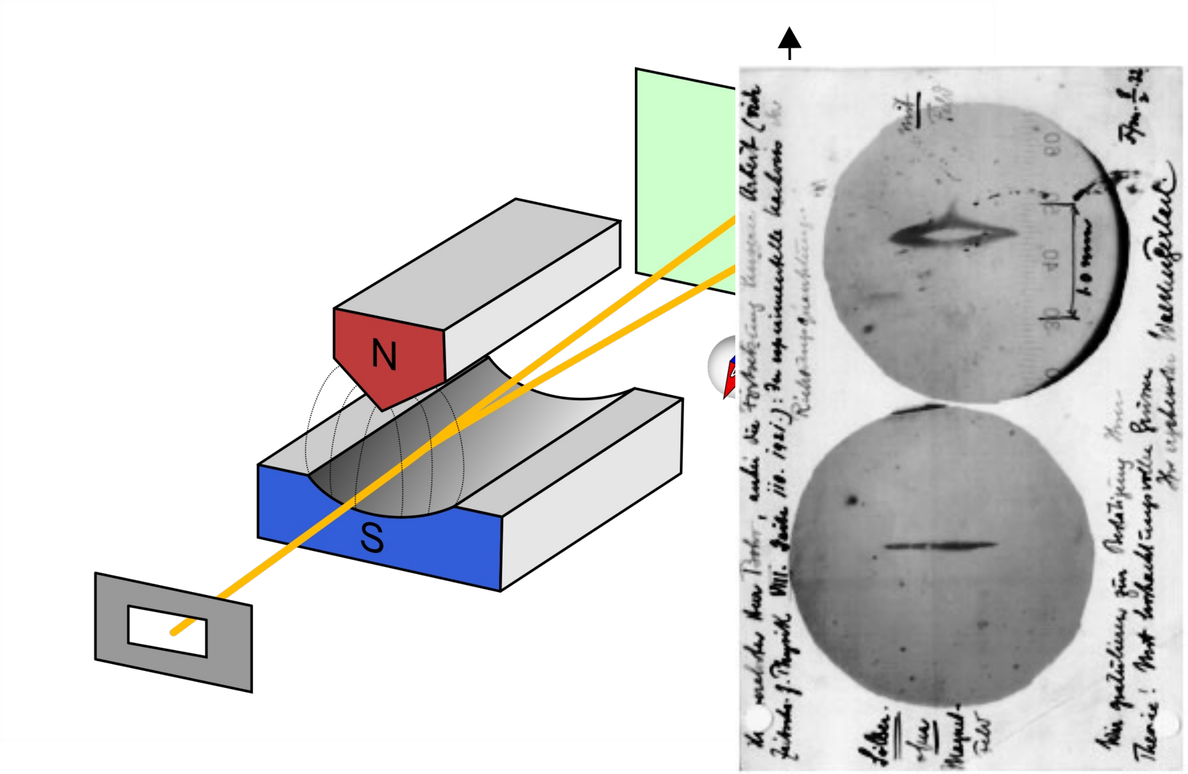

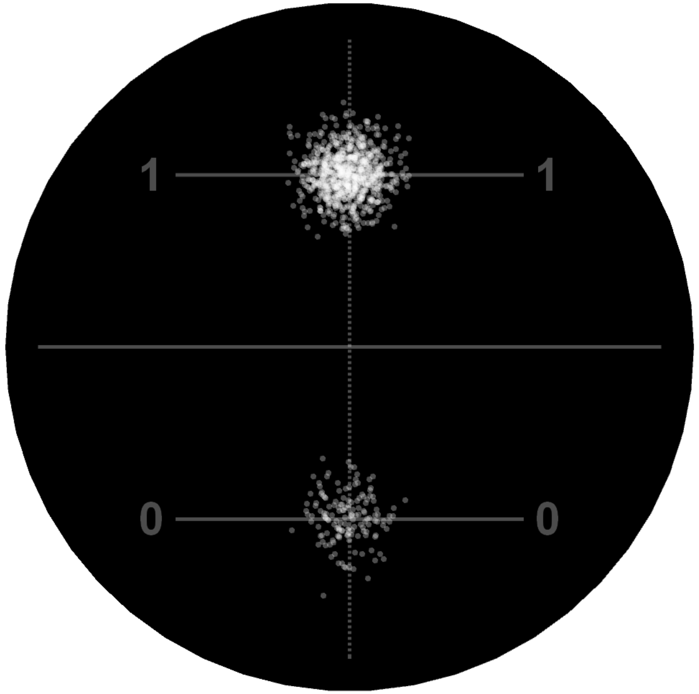

Measurements

- Measurement on part of the system is given by (Hermitian) observable **A** an the average value we observe is given by formula

$$\langle \mathbf{A} \rangle = \langle \psi | \mathbf{A} | \psi \rangle = \mathrm{Tr}[\mathbf{A} | \psi \rangle \langle \psi |]$$

- Alternatively we can use spectral decomposition of the observable

$$\mathbf{A} = \sum_j a_j |j\rangle\langle j|$$

- Measurement of A in the general state $|\psi\rangle$ yields result a_j with probability given by Born rule

$$p(j) = \mathrm{Tr}[|j\rangle\langle j| \cdot |\psi\rangle\langle\psi|] = |\langle j|\psi\rangle|^2$$

- The post-measurement state is $|j\rangle$: measurement problem – “collapse” of the state
- Heisenberg uncertainty relations – not all measurements are possible to be performed together

Qubit measurements

- Measurement is always in some basis, e.g. $|0\rangle,\ |1\rangle$ basis for observable

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Stochastic! The probability to get result

$p(0) = \|\langle 0|\psi\rangle\|^2 = \cos^2(\theta/2)$ with the state being $|0\rangle$
$p(1) = \|\langle 1|\psi\rangle\|^2 = \sin^2(\theta/2)$ with the state being $|1\rangle$

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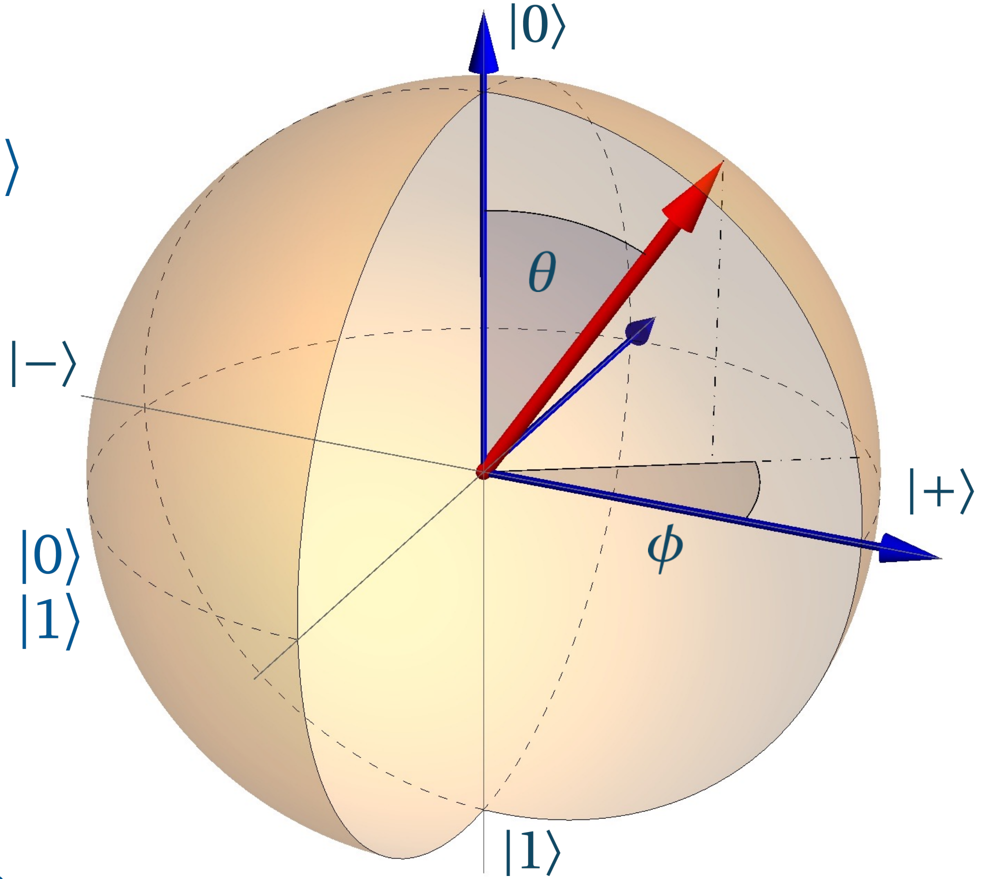
- Usually, we do not have the luxury of having a state multiple times – we can get only limited information about it



$$|\psi\rangle = \cos\frac{\theta}{2} e^{-i\frac{\phi}{2}}|0\rangle + \sin\frac{\theta}{2} e^{i\frac{\phi}{2}}|1\rangle$$

Qubit measurements

- Measurement is always in some basis, e.g. $|+\rangle, \; |-\rangle$ basis for observable

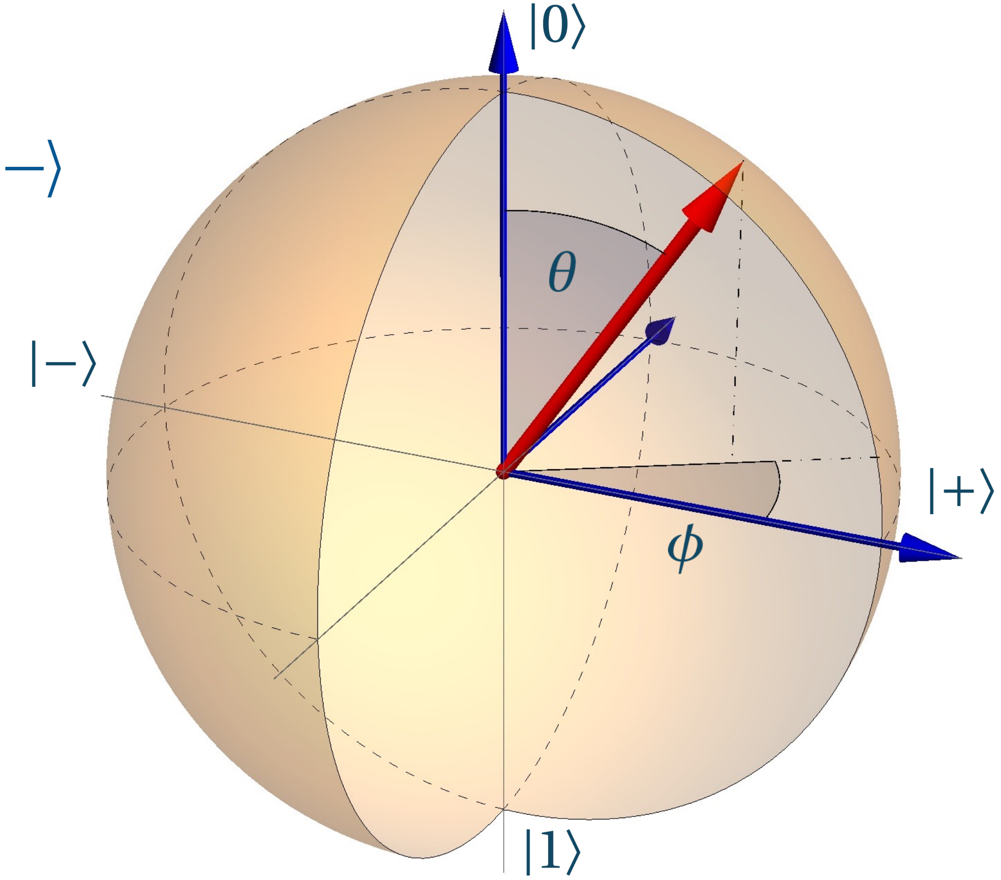
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- Stochastic! The probability to get result

$$p(\pm) = \| \langle \pm | \psi \rangle \|^2 = \frac{1}{2}(1 \pm \sin\theta \cos\phi)$$

with the state being $|\pm\rangle$

+-++-+++-+++++-++++-++-++++++-+++-+-++



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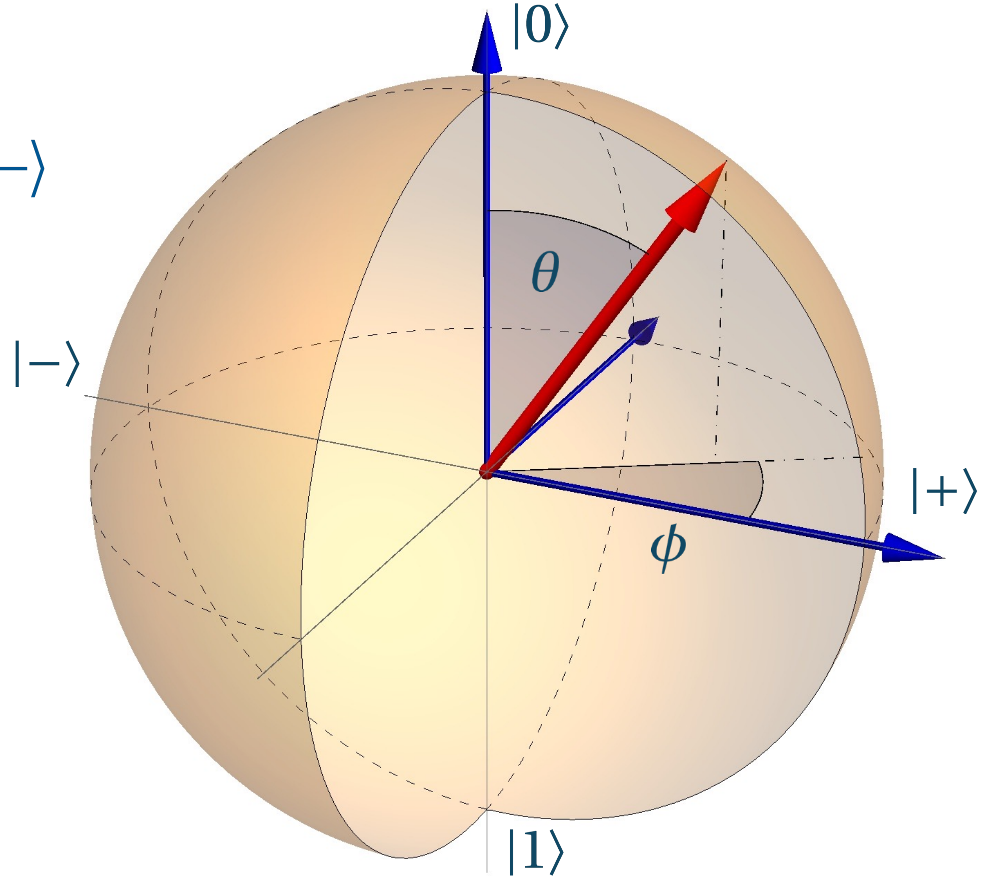
+-++-+++-+++++-++++-++-++++++-+++-+-++

- If we measured + and do now measurement in 0, 1 basis, the results will be completely random irrespective of what $|\psi\rangle$was



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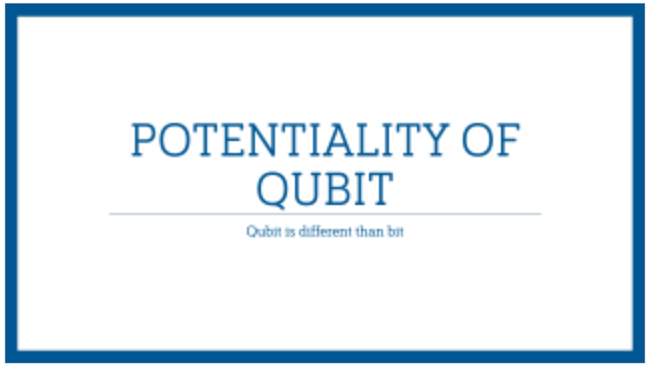
What we know so far

- State is the most complete description we have of the quantum system
- It is more complex than a classical state
- State changes are reversible, described by unitaries
- Measurements give random results and ”collapse” the state
- We cannot get all the information from the state by measuring it



TWO QUBITS

...are better than one

Bipartite systems

- Hilbert space of a composite system A-B is the tensor product $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$
- If system A is prepared in state $|\psi\rangle_A$ and system B in state $|\phi\rangle_B$ the composite state is $|\psi\rangle_A \otimes |\phi\rangle_B$
- The states $|i,\mu\rangle_{AB} \equiv |i\rangle_A \otimes |\mu\rangle_B$ where $\{|i\rangle_A\}$ and $\{|\mu\rangle_B\}$ are basis states for systems A and B, form orthonormal basis of the composite system with inner product

$${}_{AB}\langle i,\mu|j,\nu\rangle_{AB} = \delta_{ij}\delta_{\mu\nu}$$

- The tensor product operator acts on subsystems separately:

$$M_A \otimes N_B|j,\nu\rangle_{AB} = M_A|j\rangle_A \otimes N_B|\mu\rangle_B$$

- It can act on one of the systems locally:

$$M_A \otimes \mathbb{1}_B|j,\nu\rangle_{AB} = M_A|j\rangle_A \otimes |\nu\rangle_B$$

$|\psi\rangle$ Alice: Earth

Bob: Andromeda galaxy $|\phi\rangle$

Measurements on bipartite systems

- Measurement on part of the system

$$\langle \mathbf{A} \rangle = {}_{AB}\langle \Psi | \mathbf{A} \otimes \mathbb{1} | \Psi \rangle_{AB} = \mathrm{Tr}[\mathbf{A} \otimes \mathbb{1} | \Psi \rangle_{AB} \langle \Psi |]$$

- Spectral decomposition of composite observable is

$$\mathbf{A} \otimes \mathbb{1} = \sum_j a_j | j \rangle \langle j | \otimes \mathbb{1} = \sum_{j,k} a_j | j \rangle \langle j | \otimes | k \rangle \langle k |$$

- The general state

$$|\Psi\rangle_{AB} = \sum_{j,k} c_{j,k} | j \rangle \otimes | k \rangle$$

- The measurement yields result a_j with probability $p(j) = \sum_k p(j,k) = \sum_k |c_{j,k}|^2$ and the post-measurement state is

$$|\psi_j\rangle_B = \frac{1}{\mathcal{N}} \sum_k c_{j,k} | k \rangle$$

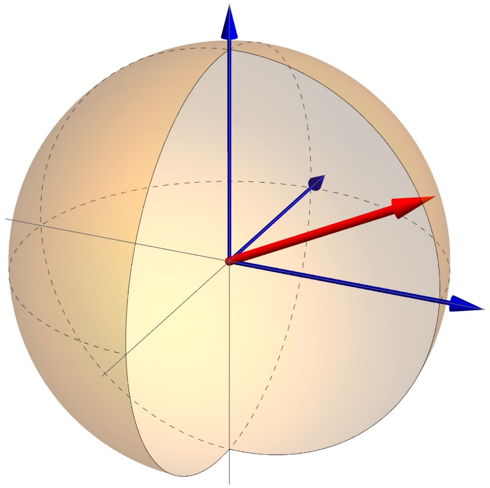
Bringing qubits together

- Two-qubit Hilbert space $\mathscr{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$
- Basis states:

$$|0\rangle \otimes |0\rangle \equiv |00\rangle \qquad |0\rangle \otimes |1\rangle \equiv |01\rangle \qquad |1\rangle \otimes |0\rangle \equiv |10\rangle \qquad |1\rangle \otimes |1\rangle \equiv |11\rangle$$

- States are no longer representable by Bloch spheres!

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \neq$$



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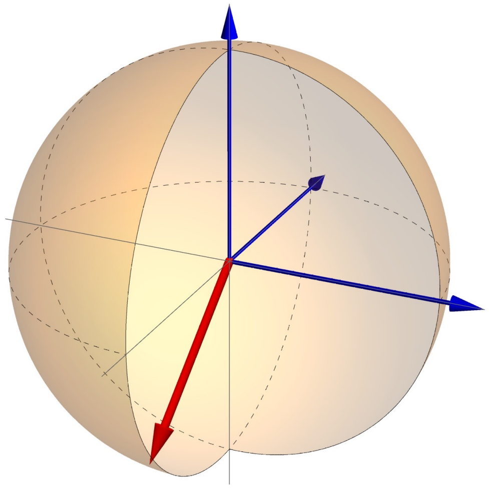

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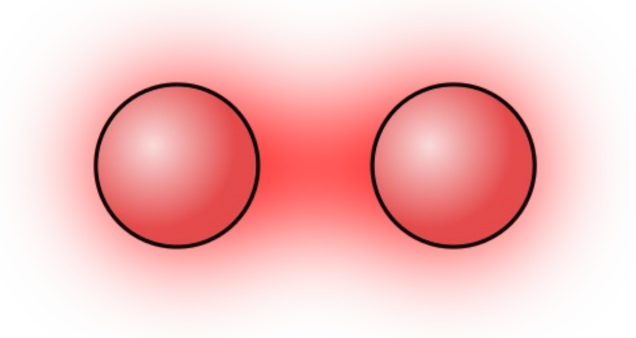
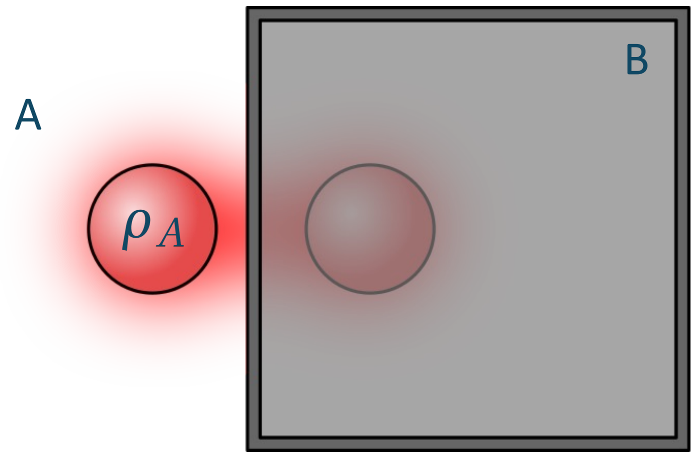
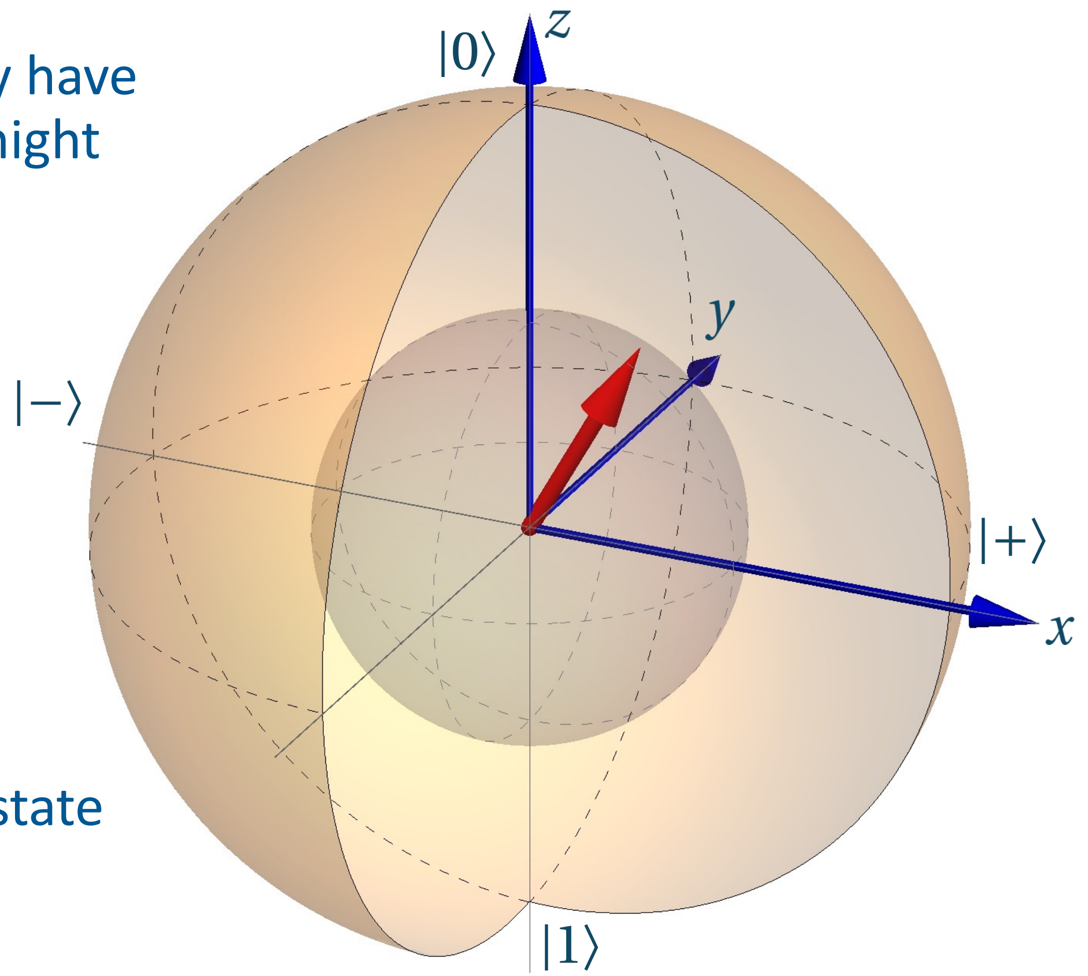


Density operator

- Having a bipartite system A-B, we might only have access to one of its parts, let's say A and B might be inaccessible

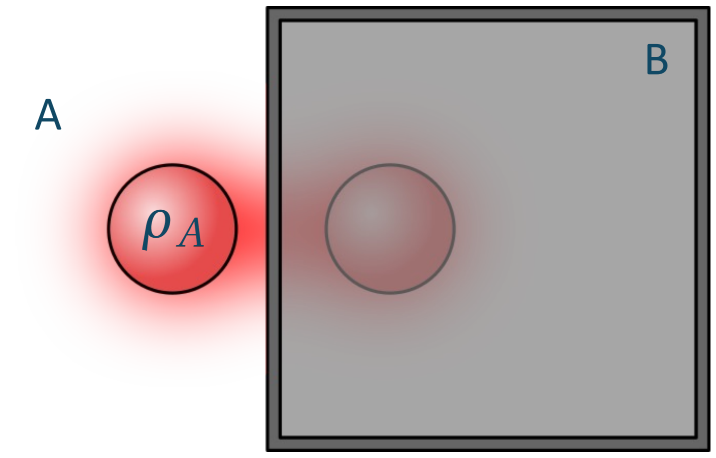




- B might be e.g. environment, or part of the state sent to Bob in Andromeda galaxy
- Our description of state is ρ_A we call it the density operator

Density operator

- Having a bipartite system A-B, we might only have access to one of its parts, let's say A and B might be inaccessible



- B might be e.g. environment, or part of the state sent to Bob in Andromeda galaxy
- Our description of state is ρ_A we call it the density operator

Density matrix is the most complete description we have of the quantum system (locally)

The density matrices together give less information than that of the joint (pure) state – where is the rest of the information?

What we know so far

- State is the most complete description we have of the quantum system
- It is more complex than a classical state
- State changes are reversible, described by unitaries
- Measurements give random results and ”collapse” the state
- We cannot get all the information from the state by measuring it
- The state of two qubits is more than just the product of the qubits’ states

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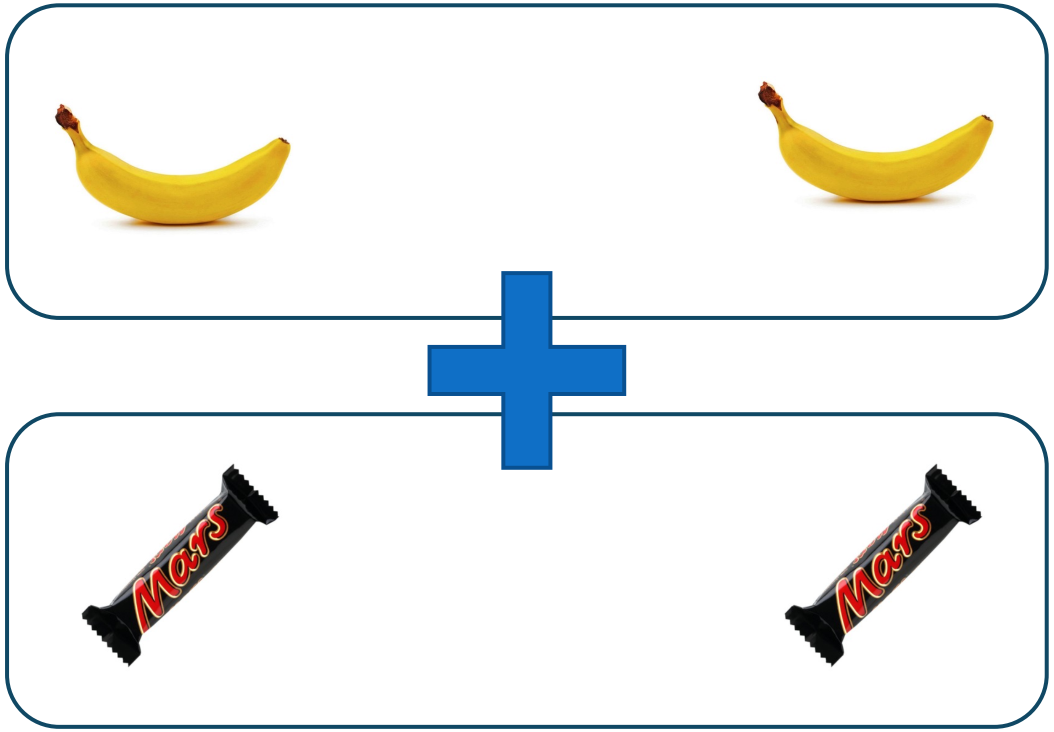
ENTANGLEMENT

EPR paradox and non-locality

Entanglement v. classical correlations

Scott Kelly

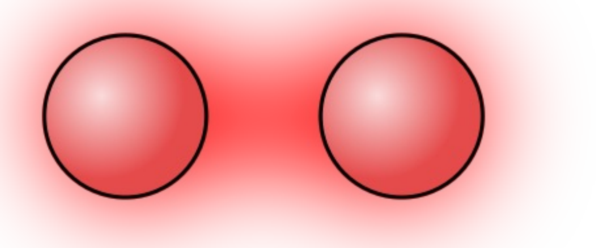


Mark Kelly

Correlations are “stronger” in the quantum case

$$|\Psi_+\rangle = \frac{1}{\sqrt{2}}\left(|\quad\rangle + |\quad\rangle\right)$$

Bell basis



- We were talking about the states $|\Psi_\pm\rangle = \dfrac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$
- These are orthogonal, but we can make a full basis of maximally entangled states by taking

$$|\Psi_+\rangle = (\mathbb{1} \otimes \mathbb{1})|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Psi_x\rangle = (\sigma_x \otimes \mathbb{1})|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$$

$$|\Psi_z\rangle = (\sigma_z \otimes \mathbb{1})|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = |\Psi_-\rangle$$

$$\boxed{|\Psi_y\rangle = (\sigma_y \otimes \mathbb{1})|\Psi_+\rangle = \frac{\mathrm{i}}{\sqrt{2}}(|10\rangle - |01\rangle)}$$

- Conversely we can write

$$|00\rangle = \frac{1}{\sqrt{2}}(|\Psi_+\rangle + |\Psi_z\rangle)$$

$$|01\rangle = \frac{1}{\sqrt{2}}(|\Psi_x\rangle + \mathrm{i}|\Psi_y\rangle)$$

$$|11\rangle = \frac{1}{\sqrt{2}}(|\Psi_+\rangle - |\Psi_z\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}}(|\Psi_x\rangle - \mathrm{i}|\Psi_y\rangle)$$

Singlet state

- All Bell states are useful, but state $|\Psi_y\rangle = \frac{\mathrm{i}}{\sqrt{2}}(|10\rangle - |01\rangle)$ has an interesting property, that it has the same form in every basis
- In particular we can also write it as

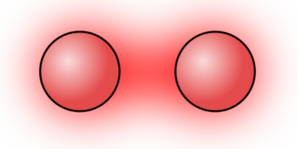
$$|\Psi_y\rangle = \frac{\mathrm{i}}{\sqrt{2}}(|-+\rangle - |+-\rangle)$$

- This state is called singlet

EPR paradox

- Einstein, Podolsky, Rosen (1935) – spooky action at a distance
- Let us have entangled state: $|\Psi_y\rangle = \frac{\mathrm{i}}{\sqrt{2}}(|10\rangle - |01\rangle)$

$|\Psi_y\rangle$



Einstein, A; B Podolsky; N Rosen – Can Quantum-Mechanical Description of Physical Reality be Considered Complete?, Phys. Rev. 47, 777–780 (1935)

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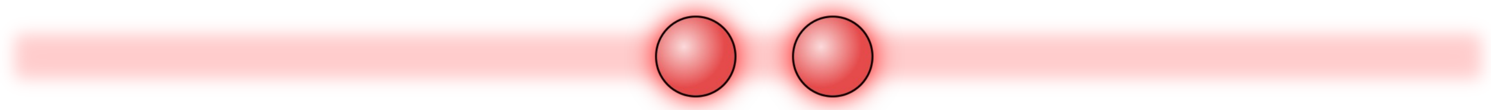
Alice: Earth

Bob: Andromeda galaxy

$|\Psi_y\rangle$



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Alice: Earth

Bob: Andromeda galaxy

- If Alice measures in $\{|0\rangle, |1\rangle\}$ basis, then if she measures 0, she immediately knows Bob will measure 1 and if she measures 1, Bob will measure 0

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Alice: Earth

Bob: Andromeda galaxy

- If Alice measures in $\{|0\rangle, |1\rangle\}$ basis, then if she measures 0, she immediately knows Bob will measure 1 and if she measures 1, Bob will measure 0
- If Alice measures in $\{|+\rangle, |-\rangle\}$ basis, then if she measures +, she immediately knows Bob will measure – and if she measures –, Bob will measure +

Einstein, A; B Podolsky; N Rosen – Can Quantum-Mechanical Description of Physical Reality be Considered Complete?, Phys. Rev. 47, 777–780 (1935)

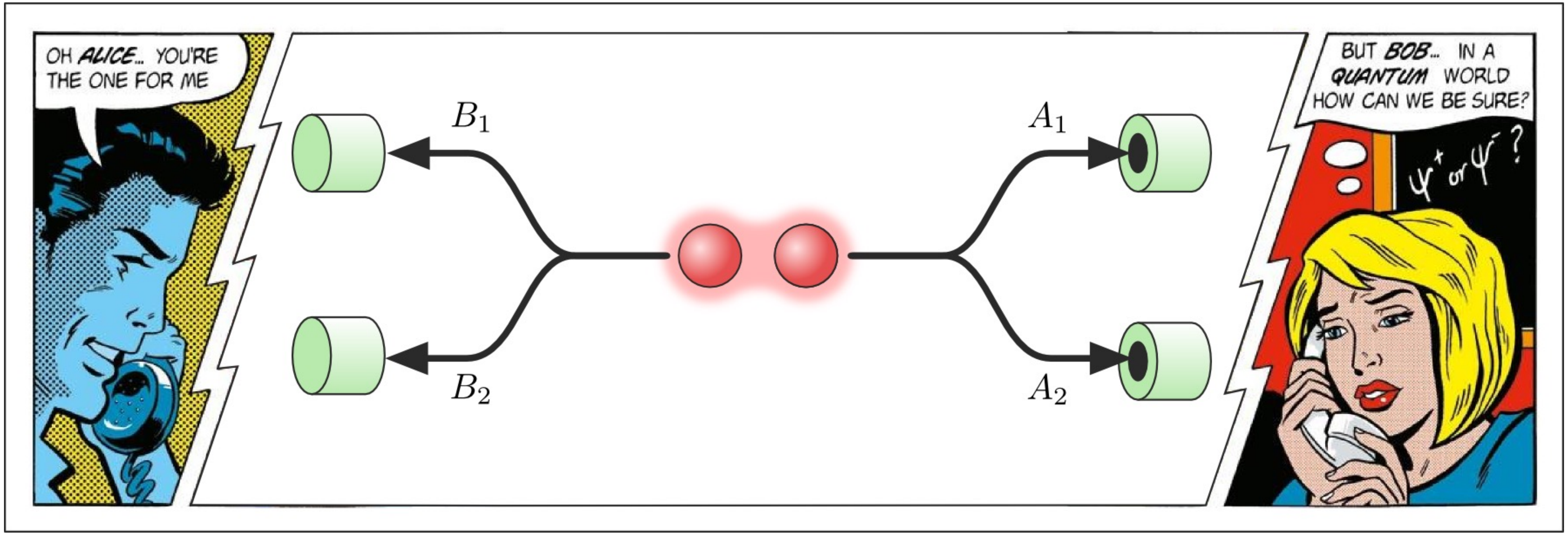
EPR paradox

- This is paradoxical because:
  - It is reasonable to assume that the measurement in a galaxy far, far away.... cannot affect our state
  - But then, together with the strong correlation, it implies, that all the measurements outcomes have to be predetermined at the point of their creation
  - This in turn means that even the possibilities ruled out by the Heisenberg uncertainty principle are somehow determined for every state
- EPR: As our description of states conforms to the uncertainty principle, it has to be incomplete
- Bell: QM is weird; this incompleteness has measurable consequences

J.S. Bell – On the Einstein Podolsky Rosen Paradox, Physics, 1: 195–200 (1964)
J.F. Clauser; M.A. Horne; A. Shimony; R.A. Holt – Proposed experiment to test local hidden-variable theories, Phys. Rev. Lett., 23: 880 (1969)

CHSH

- Measurements A and B have two-outcomes $a, b \in \{-1, +1\}$
- They both gather statistics on their joint probability $P(a, b | A_i, B_j)$



(C) John Richardson

CHSH and local realism

- From these probabilities they can compute correlation functions

$$C(A_i, B_j) = \sum_{a,b} ab P(a, b | A_i, B_j)$$

- Then they compute

$$\mathrm{B} = C(A_1, B_1) + C(A_1, B_2) + C(A_2, B_1) - C(A_2, B_2)$$

- It looks reasonable to assume (but do not take it for granted!) that the system obtains its properties during the preparation and these properties for each system determine what the results of different results will be (local realism):

$$P(a, b | A_i, B_j) = \sum_{\lambda} P(a | A_i, \lambda) P(b | B_j, \lambda) P(\lambda)$$

Alice: Earth

Bob: Andromeda galaxy

CHSH and local realism

- Let us compute $\mathrm{B} = C(A_1, B_1) + C(A_1, B_2) + C(A_2, B_1) - C(A_2, B_2)$
- Under local realism $C(A_i, B_j) = \sum_{\lambda} \langle A_i \rangle_\lambda \langle B_j \rangle_\lambda P(\lambda)$
  where

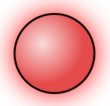
$$\langle A_i \rangle_\lambda = \sum_a a P(a|A_i, \lambda) \qquad \langle B_j \rangle_\lambda = \sum_b b P(b|B_j, \lambda)$$

- Then

$$\mathrm{B} = \sum_\lambda [\langle A_1 \rangle_\lambda \langle B_1 \rangle_\lambda + \langle A_1 \rangle_\lambda \langle B_2 \rangle_\lambda + \langle A_2 \rangle_\lambda \langle B_1 \rangle_\lambda - \langle A_2 \rangle_\lambda \langle B_2 \rangle_\lambda] P(\lambda) \leq 2$$

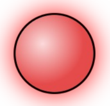
- And so (CHSH inequality)

$$\mathrm{B} = C(A_1, B_1) + C(A_1, B_2) + C(A_2, B_1) - C(A_2, B_2) \leq 2$$



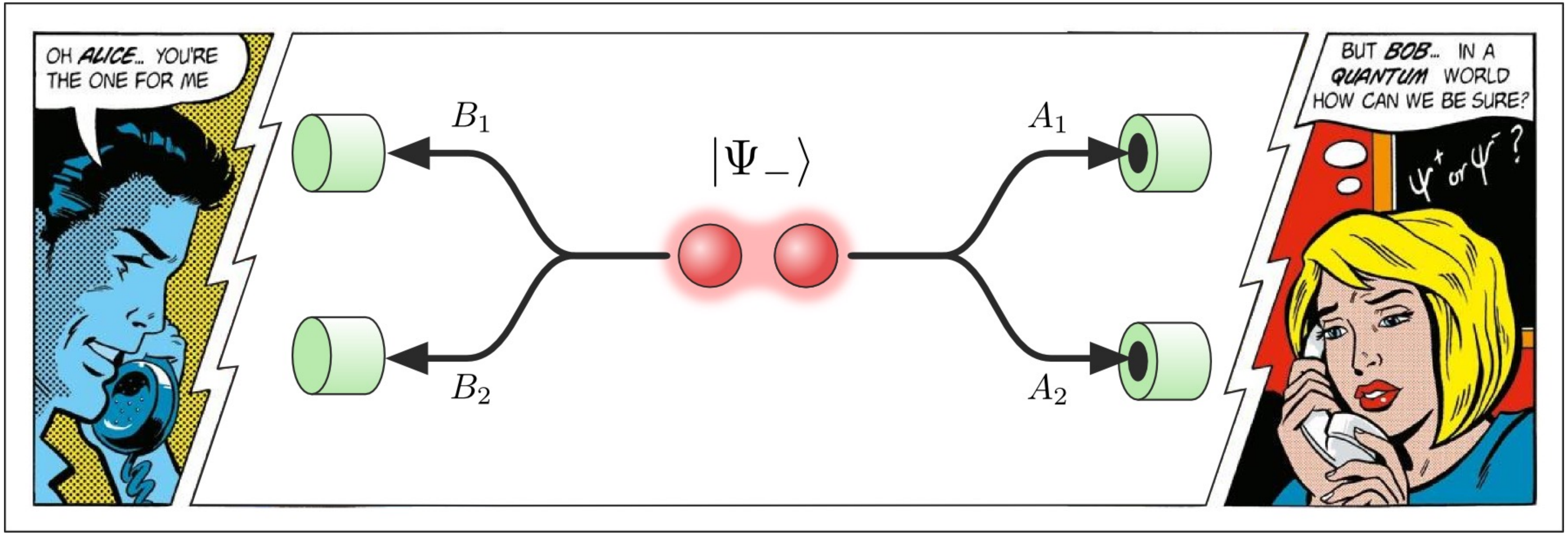
Alice: Earth

Bob: Andromeda galaxy



CHSH in the quantum case

- Now let Alice and Bob share state $|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$



(C) John Richardson

CHSH in the quantum case

- Now let Alice and Bob share state $|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
- We now have $C(A_i, B_j) = \langle\Psi_-|\mathbf{A}_i \otimes \mathbf{B}_j|\Psi_-\rangle$
- Taking (check the outcomes)

$$\mathbf{A}_1 = \sigma_x \qquad \mathbf{A}_2 = \sigma_z \qquad \mathbf{B}_1 = \frac{1}{\sqrt{2}}(\sigma_z - \sigma_x) \qquad \mathbf{B}_2 = -\frac{1}{\sqrt{2}}(\sigma_z + \sigma_x) = -H$$

  we find:

$$C(A_1, B_1) = C(A_2, B_1) = C(A_1, B_2) = -C(A_2, B_2) = \frac{1}{\sqrt{2}}$$

Non-locality

- The CHSH inequality is thus violated: $\mathrm{B} = 2\sqrt{2} > 2$
- This violation is due to quantum correlations being different from classical
- The violation of $2\sqrt{2}$ is maximal (Tsirelʹson bound)

EPR paradox (continued)

- Einstein, Podolsky, Rosen (1935) – spooky action at a distance
- Let us have entangled state: $|\Psi_y\rangle = \frac{\mathrm{i}}{\sqrt{2}}(|-+\rangle - |+-\rangle)$

Alice: Earth

$|1\rangle$

Bob: Andromeda galaxy

$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$

- If Alice measures in $\{|0\rangle, |1\rangle\}$ basis, then whatever she measures, if Bob will decide to measure in the $\{|+\rangle, |-\rangle\}$ basis he will be getting the two basis states with equal probability
- But he would be getting those even if Alice would measure in the $\{|+\rangle, |-\rangle\}$ basis (why?)
- So they can reveal the “paradox” only after communicating – no FTL communication

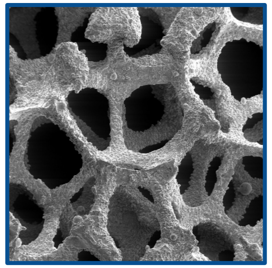
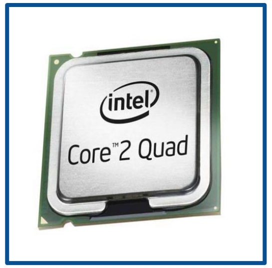
What we know so far

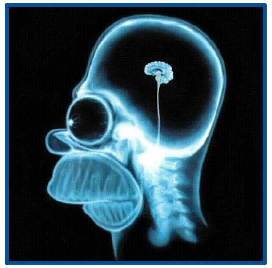
- State is the most complete description we have of the quantum system
- It is more complex than a classical state
- State changes are reversible, described by unitaries
- Measurements give random results and ”collapse” the state
- We cannot get all the information from the state by measuring it
- The state of two qubits is more than just the product of the qubits’ states
- Quantum theory is non-local

USEFULNESS OF QUANTUM INFORMATION

Second quantum revolution

- First quantum revolution (20th century): understanding the laws of quantum theory and finding direct uses of observed phenomena
- Second quantum revolution (now): using quantum theory to prepare conditions for manipulation and targeted use of quantum systems


Quantum computation

Quantum assisted communication

Quantum sensing

Possibilities of Quantum computers

- Shor's algorithm and other algorithmic applications.
- Material design

and therefore full attention and
phenomena—the challenge of exp
—has to be put into the argument, and
be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.

9. DISCUSSION

Question: Just to interpret, you spoke first of the probability of A given

R. Feynman, Simulating Physics with Computers, International Journal of Theoretical Physics 21, 467 (1982)

Possibilities of Quantum computers

- Shor’s algorithm and other algorithmic applications.
- Material design
- Drug invention and chemical compounds
- Batteries
- Noisy Intermediate-Scale Quantum technologies (NISQ): simulators

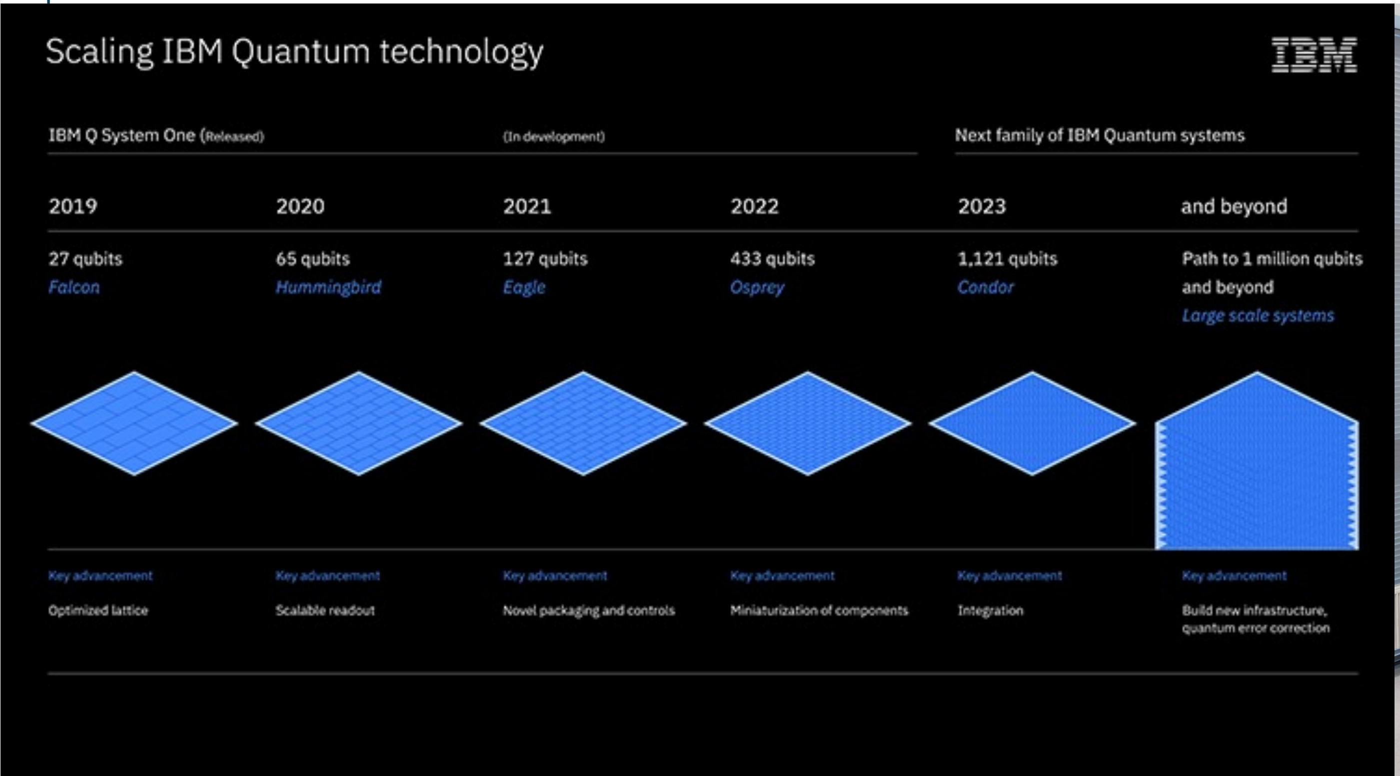
Developing a new drug and bringing it to market can take 15 years and cost more than $ 1 billion; simulations maximize commercial potential, while reducing the costs and minimizing risks

Computer Michael @ University College London
£ 1.6M, 265 TFLOPS
Just for car battery simulations

R. Feynman, Simulating Physics with Computers, International Journal of Theoretical Physics 21, 467 (1982)

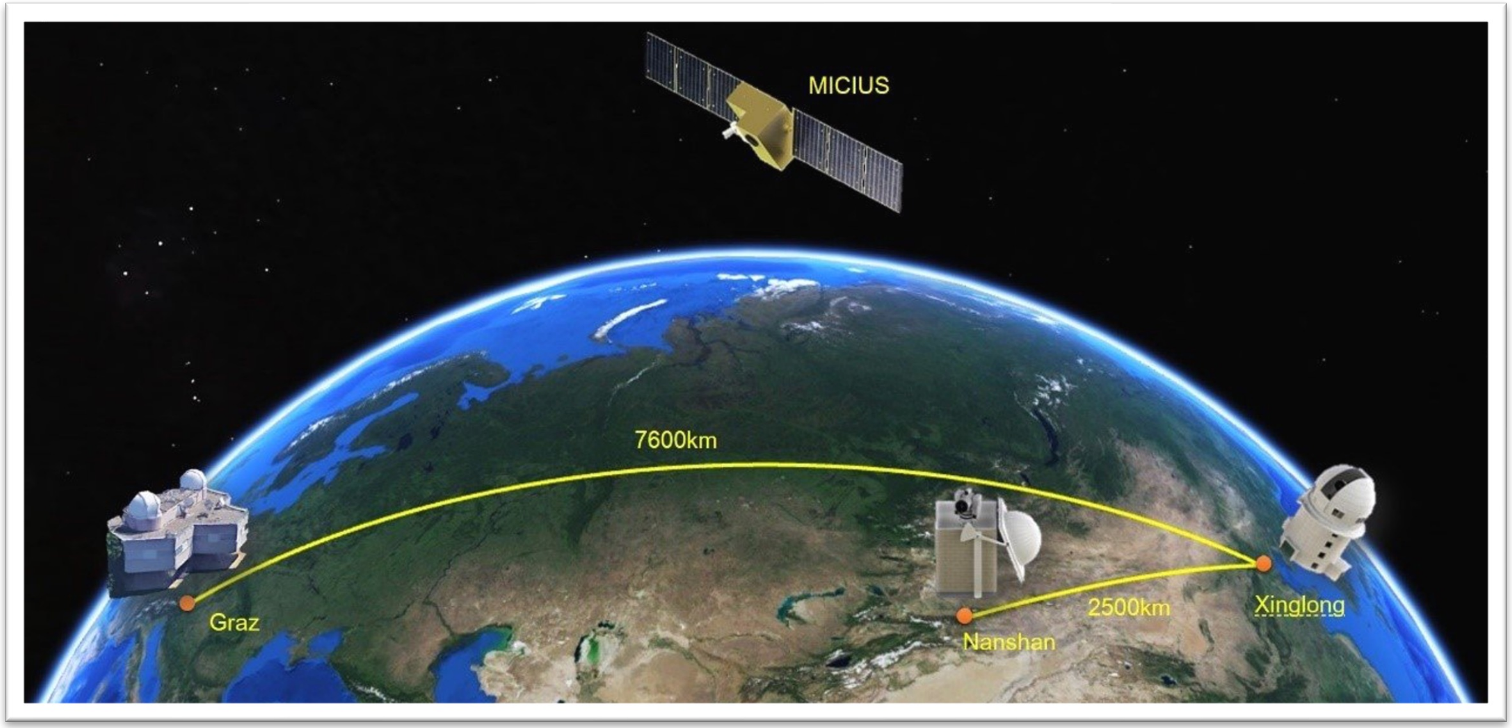
Current state-of-the-art QPUs

- Many companies working on a universal QPU: Honeywell, D-Wave, IBM, Google, Microsoft, IonQ, Rigetti, IQM
- Quantum supremacy?



QKD in practice

- Increased security
- Well developed theory
- Technological accessibility
- From quantum links
  to quantum internet
- Local quantum networks:
  - DARPA, USA
  - SECOQC, Vienna
  - SwissQuantum, Geneva
  - Tokyo QKD Network
  - LANL (USA)
- China: Beijing-Shanghai, MICIUS



What we learnt (Conclusion)

- State is the most complete description we have of the quantum system
- It is more complex than a classical state
- State changes are reversible, described by unitaries
- Measurements give random results and ”collapse” the state
- We cannot get all the information from the state by measuring it
- The state of two qubits is more than just the product of the qubits’ states
- Quantum theory is non-local
- Offers possibilities for faster computation or more secure communication

beyond
the obvious

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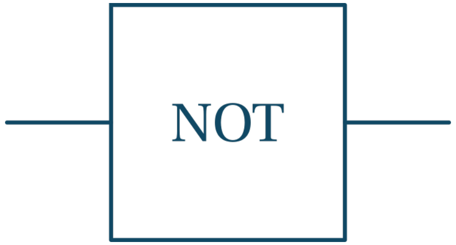
POTENTIALITY OF QUBIT

Qubit is different than bit

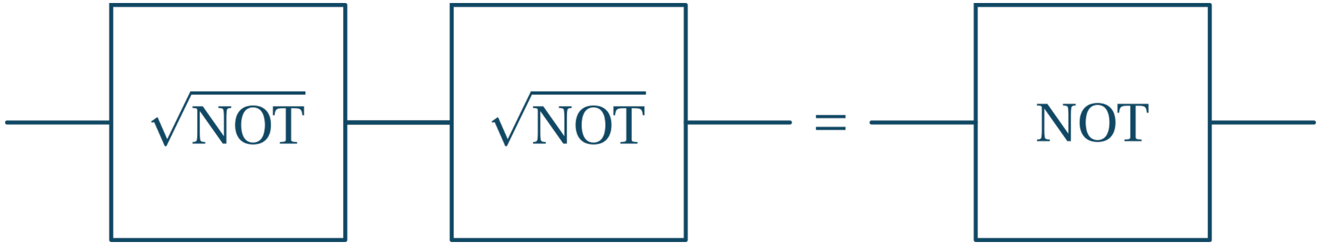
Can you do $\sqrt{\mathrm{NOT}}$?

- NOT:



$\mathrm{NOT}\ x = 1 - x$

- $\sqrt{\mathrm{NOT}}$:



- Classically clearly impossible. What about probabilistically?

D. Deutsch, A. Ekert, R. Lupacchini - Machines, Logic and Quantum Physics, The Bulletin of Symbolic Logic
Vol. 6, No. 3 (Sep., 2000), pp. 265-283; arXiv:math/9911150 [math.HO]

Can you do $\sqrt{\mathrm{NOT}}$ probabilistically?

- From bit to p-bit: $n = (p, 1-p)$ which means that $p(0) = p$ and $p(1) = 1 - p$
- Any transformation M is a **stochastic matrix**: $n_{t+1} = M n_t$
- We have $M_{\mathrm{NOT}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and we want $M_{\sqrt{\mathrm{NOT}}} = \begin{pmatrix} m_{00} & m_{01} \\ m_{10} & m_{11} \end{pmatrix} = ?$ such that $M_{\mathrm{NOT}} = M^2_{\sqrt{\mathrm{NOT}}}$
- Conditions:

$$m_{00}^2 + m_{01} m_{10} = 0 \qquad m_{00} m_{01} + m_{11} m_{01} = 1$$

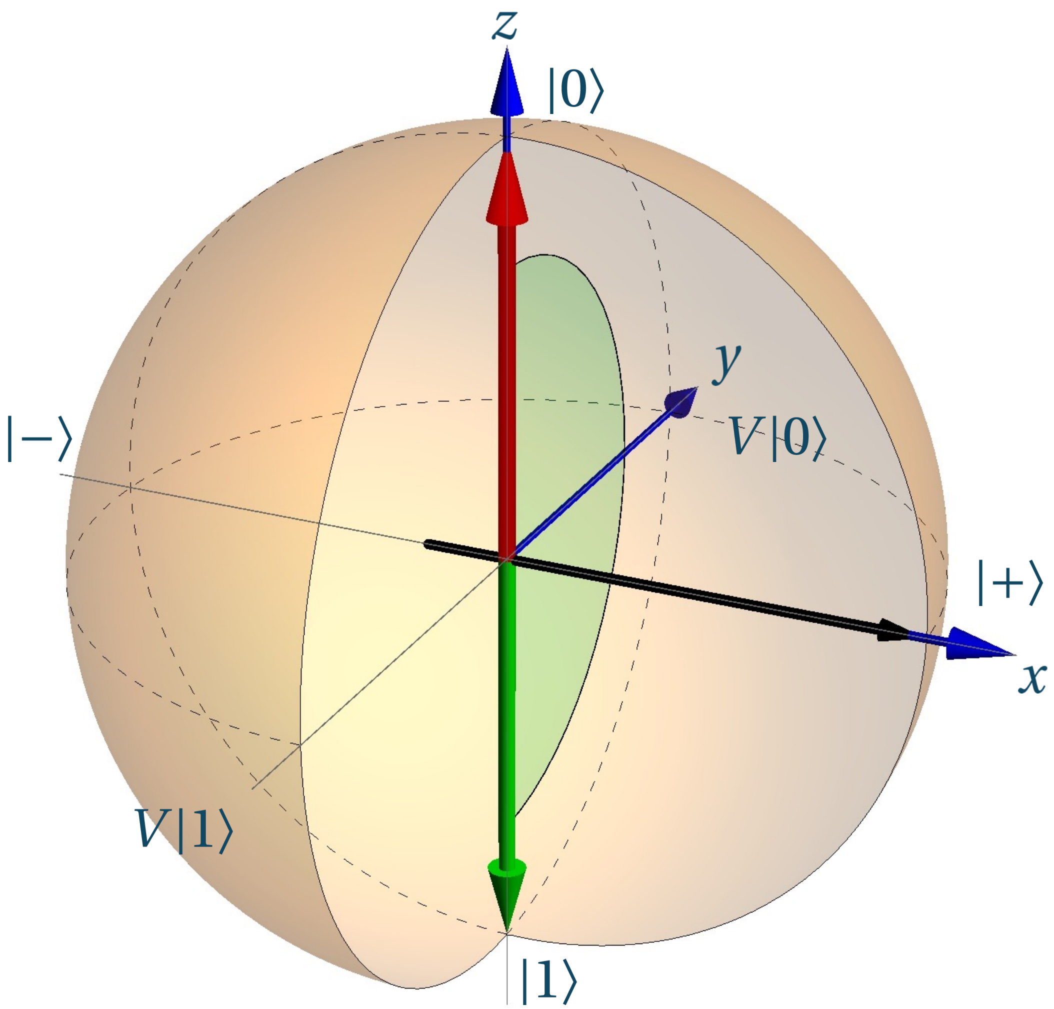
$$m_{11}^2 + m_{01} m_{10} = 0 \qquad m_{00} m_{10} + m_{11} m_{10} = 1$$

- We cannot do $\sqrt{\mathrm{NOT}}$ even probabilistically; now let us look into the quantum case

Quantum $\sqrt{\text{NOT}}$

- Similarly as qubits, we can express also qubit evolutions in Bloch representation (up to a phase):

$$U(\hat{n},\omega) = \mathrm{e}^{-\mathrm{i}\frac{\omega}{2}\hat{n}\cdot\vec{\sigma}} = \mathbb{1}\cos\frac{\omega}{2} - \mathrm{i}\hat{n}\cdot\vec{\sigma}\sin\frac{\omega}{2}$$



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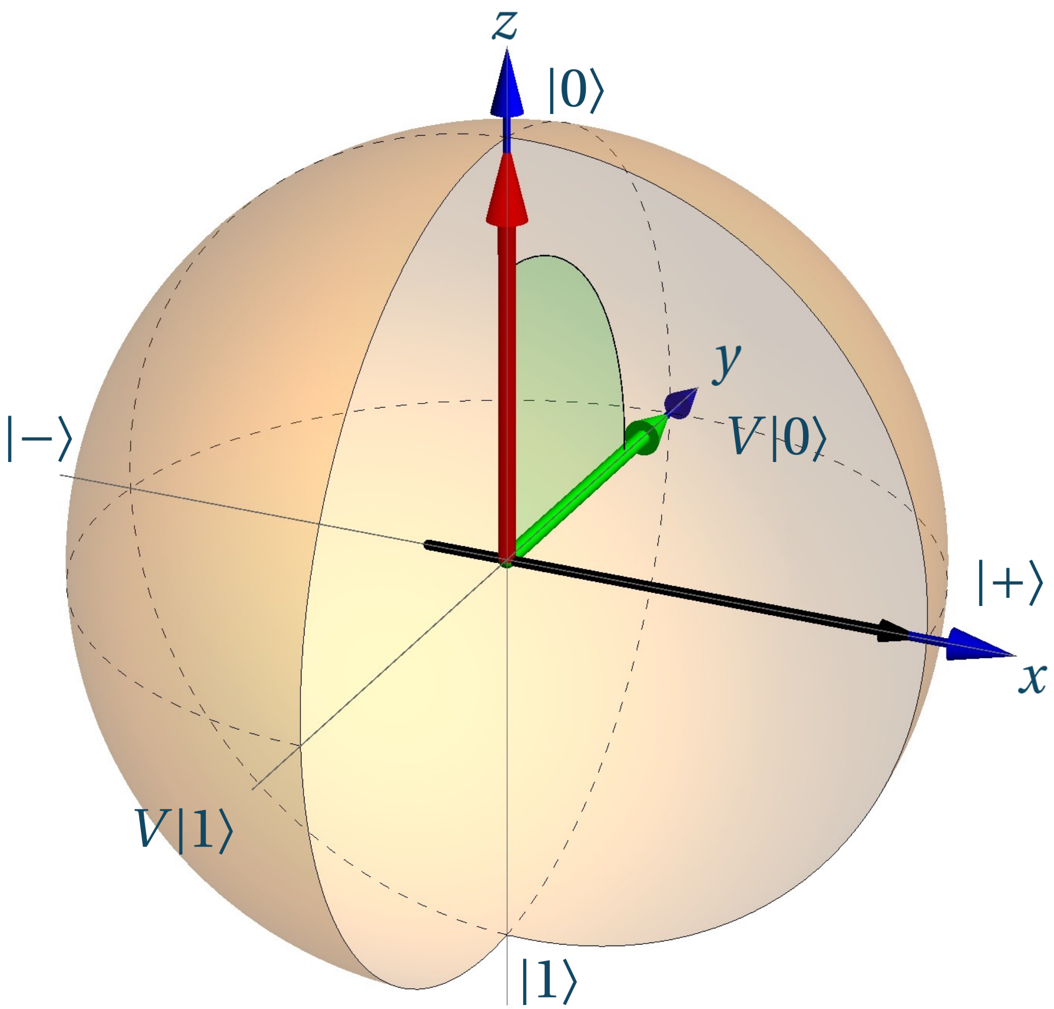
- Our square root of NOT is:

$$V = U(\hat{e}_x,\pi/2) = \sqrt{\mathrm{NOT}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}$$

- If we apply V twice we indeed get:

$$\sigma_x = U(\hat{e}_x,\pi) = \mathrm{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



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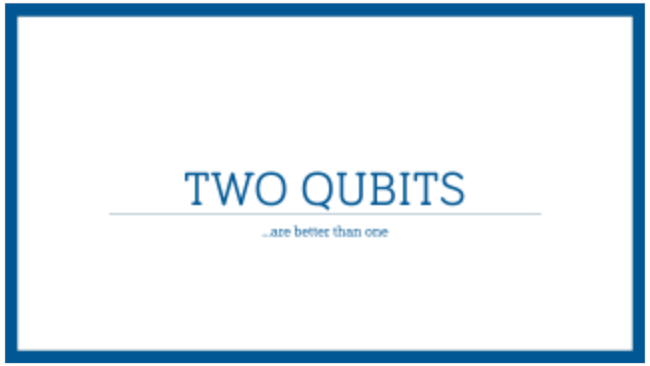

QUBIT REGISTERS

And their power…

Bringing qubits together

- Two-qubit Hilbert space $\mathscr{H} = \bigotimes^{d} \mathbb{C}^2$
- Basis states:

$$|0\rangle \otimes \ldots \otimes |0\rangle \otimes |0\rangle \equiv |0\ldots 00\rangle \qquad |0\rangle \otimes \ldots \otimes |0\rangle \otimes |1\rangle \equiv |0\ldots 01\rangle$$
$$|0\rangle \otimes \ldots \otimes |0\rangle \otimes |1\rangle \equiv |0\ldots 10\rangle \qquad |0\rangle \otimes \ldots \otimes |1\rangle \otimes |1\rangle \equiv |0\ldots 11\rangle$$
$$\ldots$$
$$|1\rangle \otimes \ldots \otimes |0\rangle \otimes |1\rangle \equiv |1\ldots 10\rangle \qquad |1\rangle \otimes \ldots \otimes |1\rangle \otimes |1\rangle \equiv |1\ldots 11\rangle$$

- States are no longer representable by Bloch spheres!
- Evolutions:

$$I \otimes \ldots \otimes V \otimes I$$

$$\boxed{U}$$

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- States are no longer representable by Bloch spheres!
- Evolutions:

$$I \otimes \ldots \otimes H \otimes I \qquad H^{\otimes d}|0\rangle^{\otimes d} = H^{\otimes d}|00\ldots 0\rangle = \frac{1}{\sqrt{2^d}} \sum_{j=0}^{2^d-1} |j_2\rangle$$

Bringing qubits together

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$$\ldots$$

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- States are no longer representable by Bloch spheres!
- Observables:

$$I \otimes \ldots \otimes P_0 \otimes I$$

M

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