FAKULTA MATEMATIKY,
FYZIKY A INFORMATIKY
Univerzita Komenského
v Bratislave

UNIVERZITA
KOMENSKÉHO
V BRATISLAVE

# Superconducting Parametric Amplifiers

S. Kern[1], **P. Neilinger[1,2]**, E. Il'ichev[3], A. Sultanov[4], M. Schmelz[3], S. Linzen[3], J. Kunert[3], G. Oelsner[3], R. Stolz[3], A. Danilov[5], S. Mahashabde[5], A. Jayaraman[5], V. Antonov[6], S. Kubatkin[5], and M. Grajcar[1,2]

*[1]Department of Experimental Physics, Comenius University, SK-84248 Bratislava, Slovakia*
*[2]Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta, Bratislava, Slovakia*
*[3]Leibniz Institute of Photonic Technology, D-07702 Jena, Germany*
*[4]Department of Applied Physics, Aalto University, P.O. Box 15100, FI-00076 Aalto, Finland*
*[5]Department of Microtechnology and Nanoscience MC2,*
*Chalmers University of Technology, SE-41296 Goteborg, Sweden*
*[6]Physics Department, Royal Holloway, University of London, Egham TW20 0EX, United Kingdom*

SAV

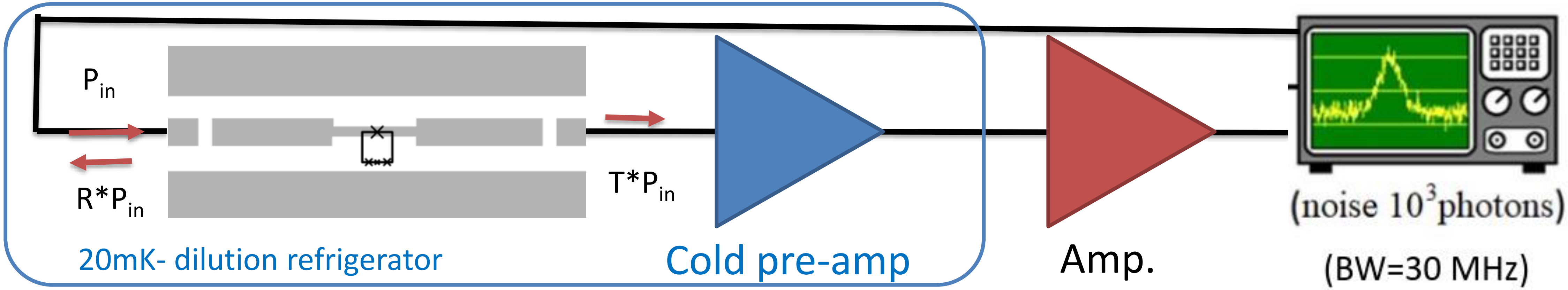
Leibniz ipht

# Experimental requirements for QC

- Circuit QED - qubit state probed by microwave signal - R,T
- Qubit energy $\Delta/h$ ~ 6 GHz = 300 mK, for high coherence $hf \gg k_b T$ → T < 50 mK
- High-Fidelity Readout $n_{cav} \leq 10$ (~ expected photons over time 1/BW in the amp.)

Quantum system, few photon signal

Room temperature electronics



$P_{in}$

$R*P_{in}$

$T*P_{in}$

20mK- dilution refrigerator

Cold pre-amp

Amp.

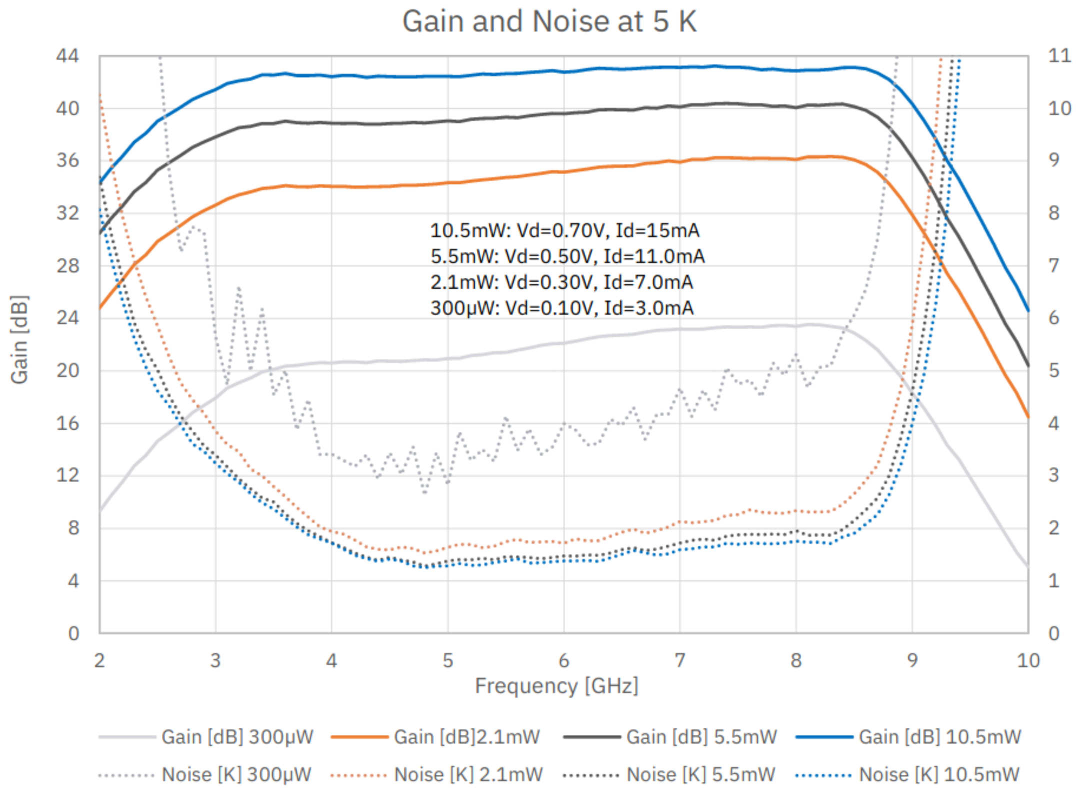
(noise $10^3$photons)

(BW=30 MHz)

- Amplifier, $T_N$ equivalent noise temperature - white noise of R=50Ω; $N_{out}=G*(N_{in}+k_b T_N)$
- High signal to noise ratio /low noise temperature is critical! Cryogenic amplifier.

# Cryogenic HEMT amplifiers

- Advantages: Broadband, high gain, flat gain profile, reliable, easy to use, high dynamic range, provide isolation, commercially available



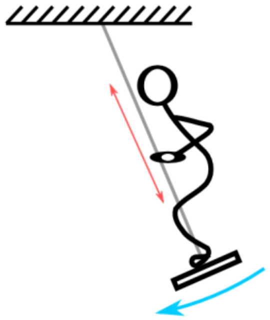
| RF Bandwidth | 4-8 GHz |
|---|---|
| Noise Temperature | 1.5-2 K |
| Gain | 42 dB |
| Power dissipation | 10 mW |

https://lownoisefactory.com/product/lnf-lnc4_8c/

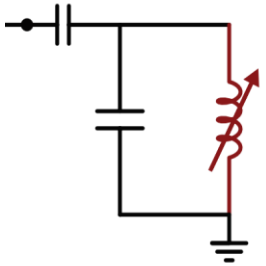
- Disadvantages: Minimum operation temperature 3-4K, high dissipation ~10mW, noise temperature 2 K → $n_{HEMT}$ ~ 10 photons

# Parametric amplification

- Driven oscillator with a modulated system parameter – energy transfer between modes
    - Child on a swing - resonance frequency $\omega_0$ modulated (pumped) at $2\omega_0$

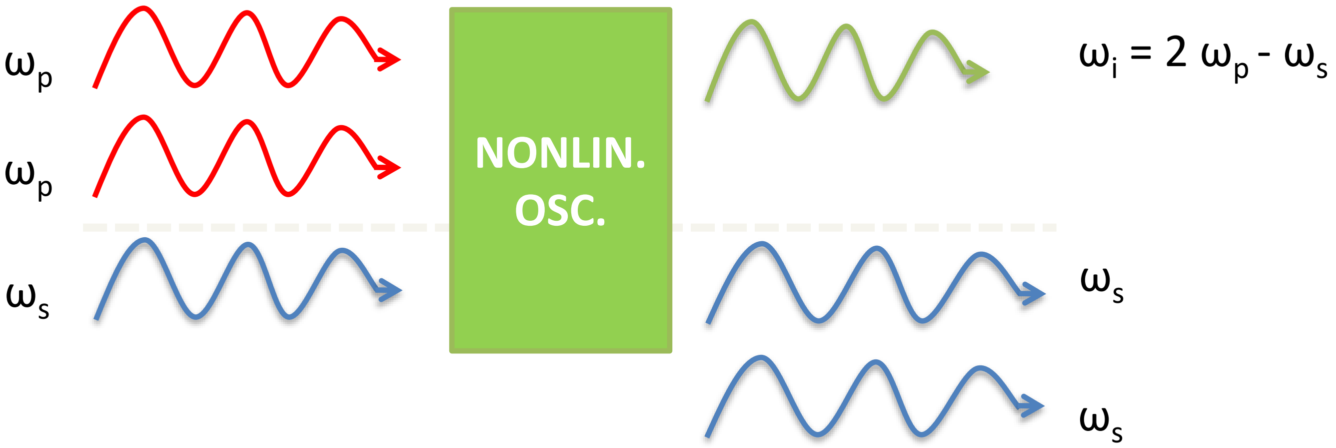


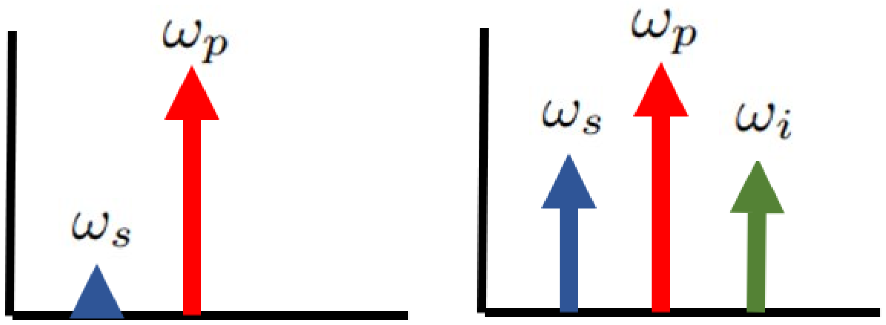
- LC resonator with variable inductance – nonlinear inductance



$L_0 + \delta L \cos(2\pi f_P t + \phi_P)$

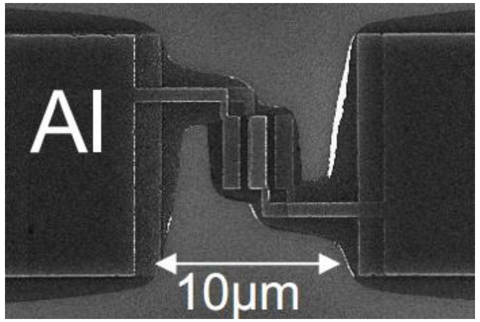
- 4 wave mixing – photon model: $\omega_p + \omega_p = \omega_s + \omega_i$





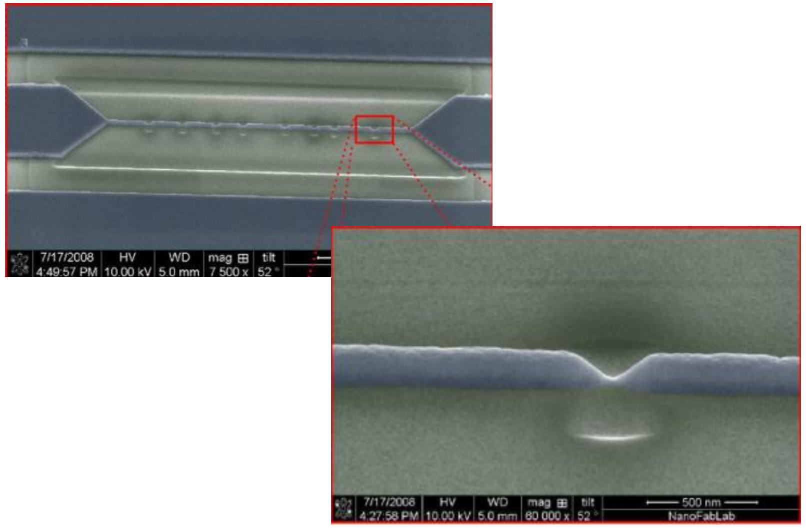
# Superconducting parametric amplifiers

- Nonlinearity – nonlinear inductance
  - Josephson Junctions and SQUIDs

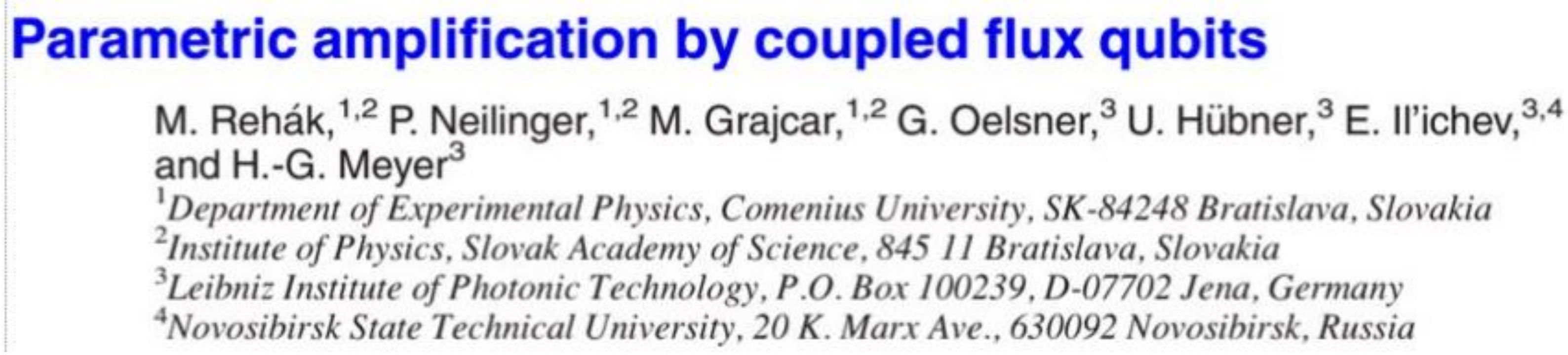


$$L(I) = \frac{\Phi_0}{2\pi I_0 \sqrt{1-(I/I_0)^2}}$$

- Disordered superconductors - High kinetic inductance



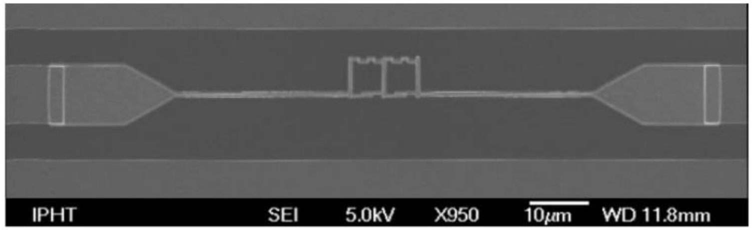
E. A. Tholén et al 2009 Phys. Scr. 2009 014019



**Parametric amplification by coupled flux qubits**

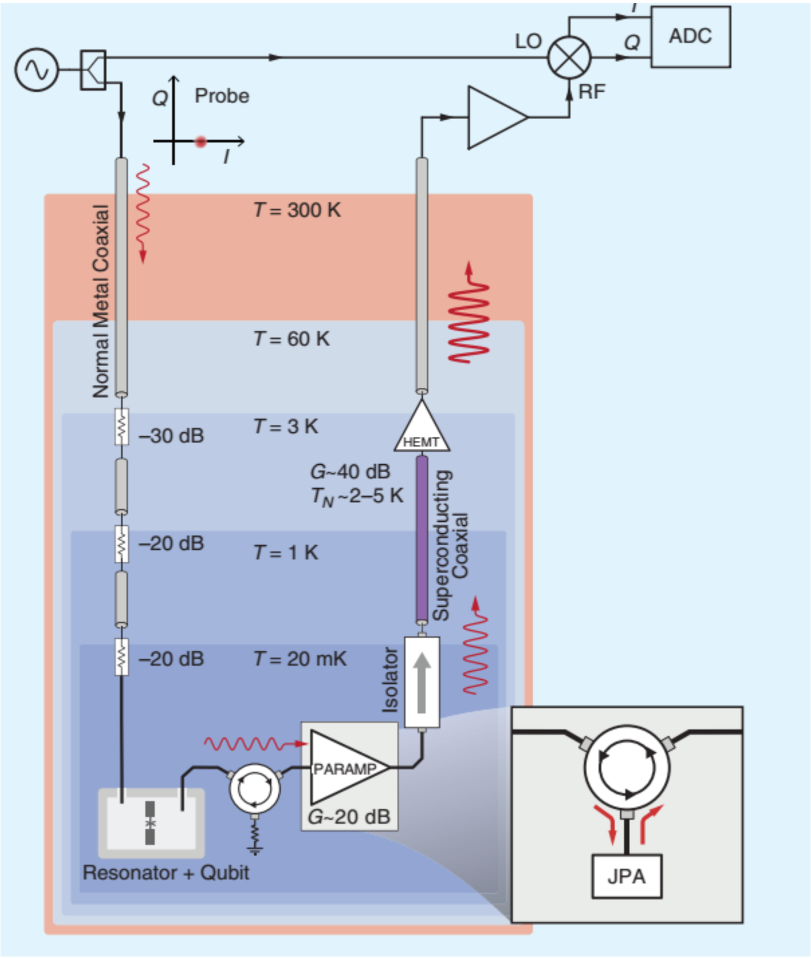
M. Rehák,[1,2] P. Neilinger,[1,2] M. Grajcar,[1,2] G. Oelsner,[3] U. Hübner,[3] E. Il'ichev,[3,4] and H.-G. Meyer[3]

[1]*Department of Experimental Physics, Comenius University, SK-84248 Bratislava, Slovakia*
[2]*Institute of Physics, Slovak Academy of Science, 845 11 Bratislava, Slovakia*
[3]*Leibniz Institute of Photonic Technology, P.O. Box 100239, D-07702 Jena, Germany*
[4]*Novosibirsk State Technical University, 20 K. Marx Ave., 630092 Novosibirsk, Russia*



# Experimental set-up for QM with ParAmps

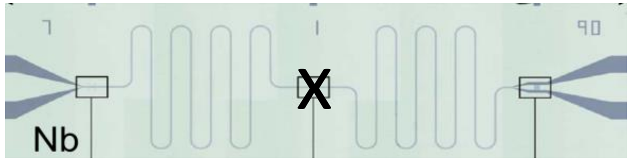
- $^3$He/$^4$He dilution refrigerator
- Paramps work at 20 mK
- Superconducting ParAmps have no dissipative elements
- Quantum limited amplifier
  - Added-noise number number $n_N^{SQL} = 1/2$
- Advantages: High gain, ultra-low noise, low power dissipation
- Superconducting ParAmps are an **enabling** technology for superconducting qubit measurement



J. Aumentado, IEEE Microwave Magazine 21(8):45-59 (2020)

# Superconducting parametric amplifiers

- Resonant parametric amplifiers

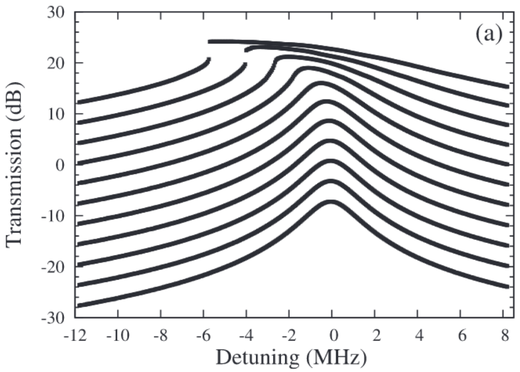


- Duffing oscillator

$$\ddot{y} + \delta\dot{y} + \alpha y + \beta y^3 = F_p\cos(\omega_p t)$$

$$y_p = P\cos(\omega_p t - \phi_p)$$

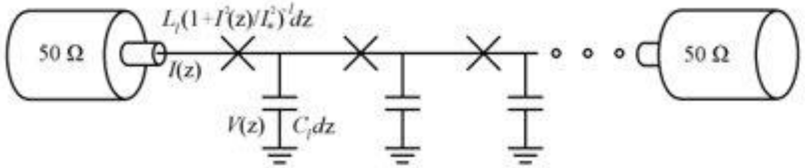
$$P = \frac{F_p}{\sqrt{(\omega_p^2 - \alpha - \frac{3}{4}\beta P^2)^2 + (\delta\omega_p)^2}}$$

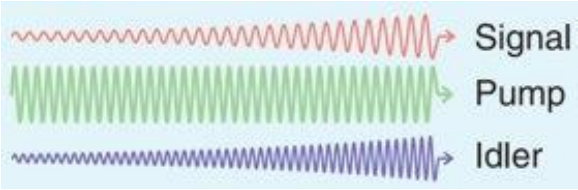


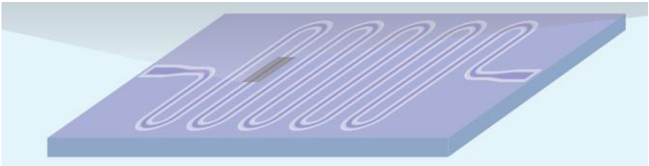
+ SQL noise and below (squeezing), high gain

- Limited bandwidth ~ 1 MHz

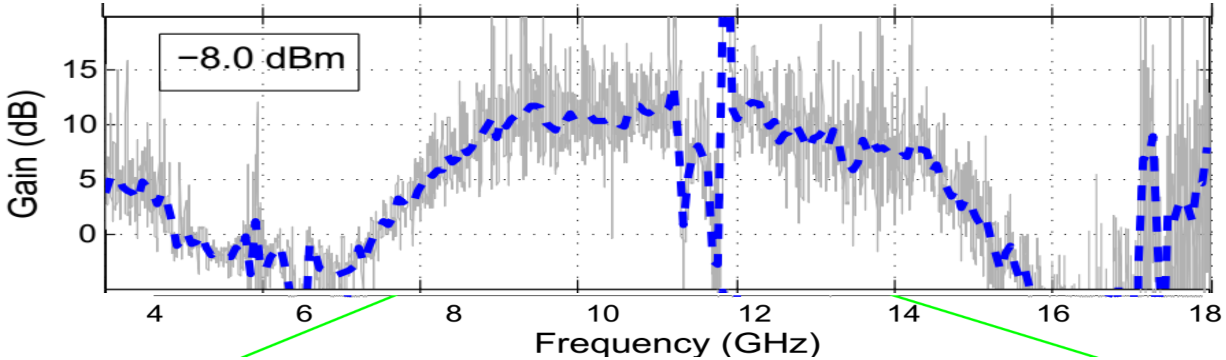
- Broadband amplifiers – traveling wave parametric amplifiers TWPAs
- Long nonlinear waveguide, array of JJs –> Wave-mixing







J. Aumentado, IEEE Microwave Magazine 21(8):45-59 (2020)



+ Broadband - several GHz

B. H. Eom et al., , Nature Physics 8, 623–627 (2012)

- Gain ripples, demanding fabrication, higher noise

# Coupled mode theory

- Current in waveguide

$$I(x,t) = \sum_{j}^{\{p,s,i\}} \frac{1}{2}\left(I_j(x)e^{i(k_j x - \omega_j t)} + c.c.\right)$$

- Telegrapher‘s equation for nonlinear medium

$$\frac{\partial^2 I(z,t)}{\partial z^2} - L_l C_l \frac{\partial^2 I(z,t)}{\partial t^2} = \frac{L_l C_l}{6 I_c^2} \frac{\partial^2 I(z,t)^3}{\partial t^2}$$

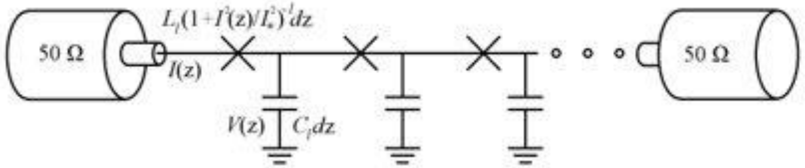
…

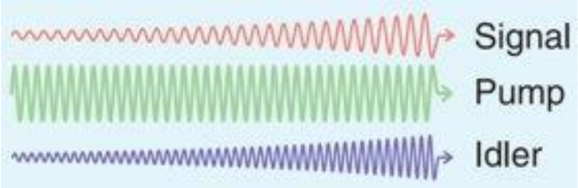
Gain: $g = \sqrt{k_s k_i \gamma^2 - \frac{\beta^2}{4}}$

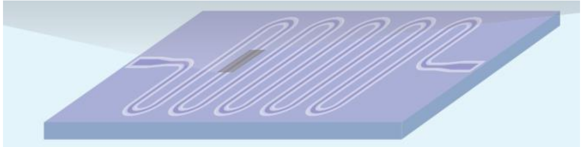
Phase mismatch: $\beta = \Delta k(1 + 2\gamma) - 2k_p\gamma$

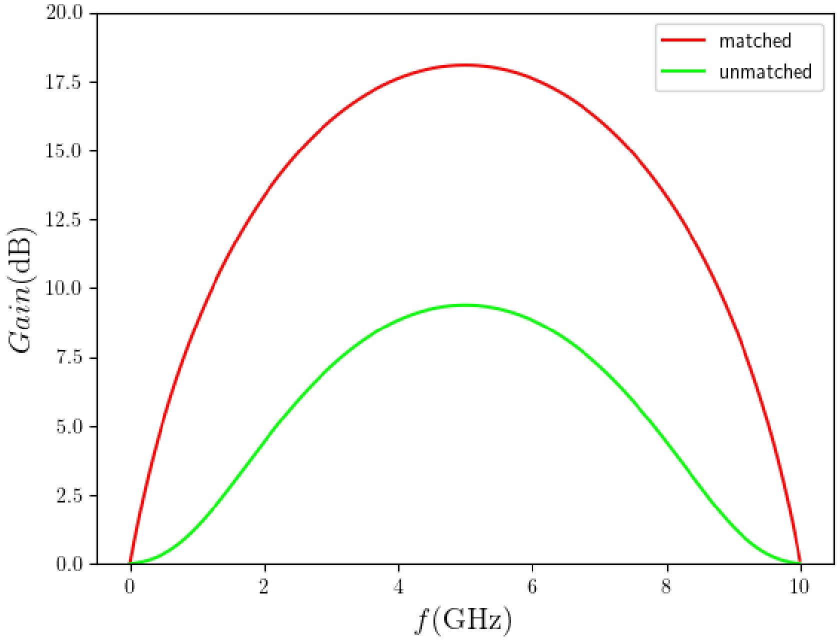
$\Delta k = 2k_p - k_s - k_i$

Nonlinearity strength: $\gamma = \frac{|t_p I_p|^2}{16 I_c^2}$









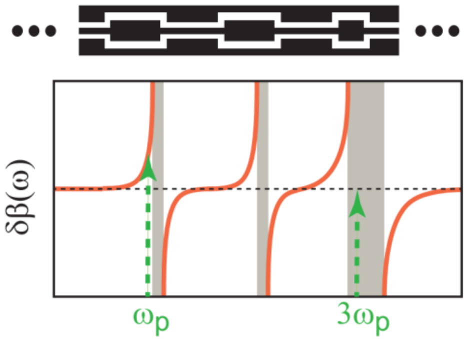
Unmatched phase β≠0:

*G ~ $l^2$*

Matched phase β=0:

*G ~ exp(l)*

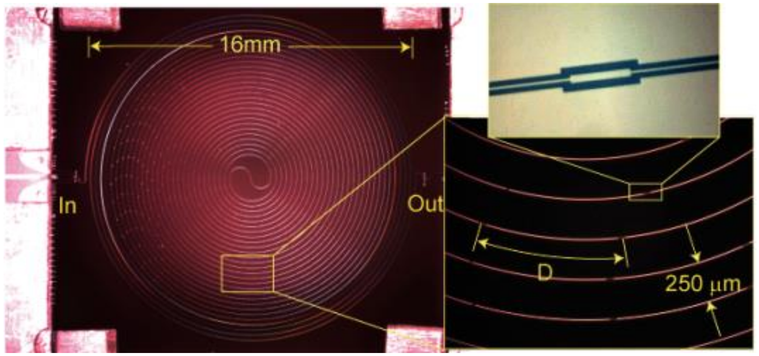
Dispersion engineering



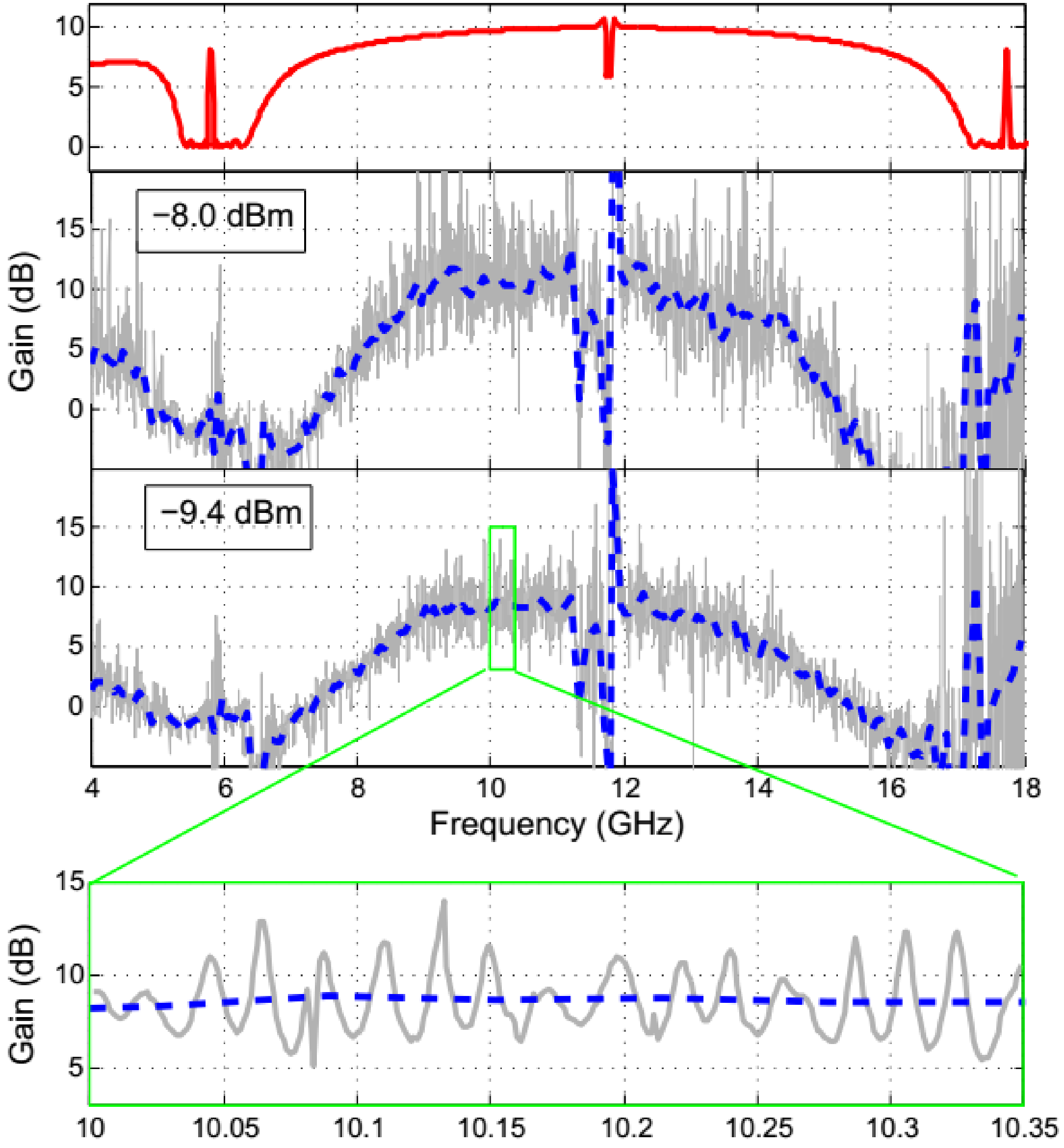
Fixed pump frequency

# Coupled mode theory vs experiment

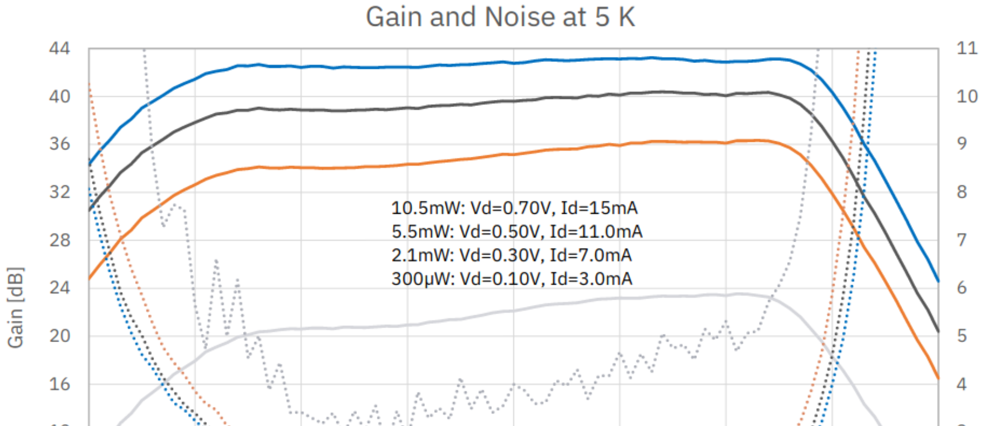
- 80 cm nonlinear waveguide, Impedance Z=200 Ω



- Phase matched – high gain
- Strong ripples - limits the usability of TWPA
- Coupled mode theory fail to explain the ripples



B. H. Eom et al., , Nature Physics 8, 623–627 (2012)



# Coupled mode theory with reflections

- Current in finite waveguide with unmatched impedance

$$I(x,t) = \sum_{j}^{\{p,s,i\}} \frac{1}{2}\Big(I_j(x) t_j (e^{ik_j x} + \Gamma e^{-ik_j x}) e^{-i\omega t} + c.c.\Big)$$

$$\frac{\partial^2 I(z,t)}{\partial z^2} - L_l C_l \frac{\partial^2 I(z,t)}{\partial t^2} = \frac{L_l C_l}{6 I_c^2} \frac{\partial^2 I(z,t)^3}{\partial t^2}$$

Gain: $$g = \sqrt{k_s k_i \gamma^2 (1 + 4|\Gamma|^2) - \frac{\beta}{4}}$$

Phase mismatch:

$$\beta = \Delta k(1 + 2\gamma(1 + \Gamma^2)) - 2k_p \gamma (1 - \Gamma^2)$$

Nonlinearity strength: $$\gamma = \frac{|t_p I_p|^2}{16 I_c^2}$$

S. Kern, et al. arXiv:2203.02448 (2022)

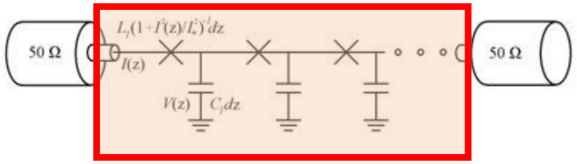
- Fabry-Perot resonator

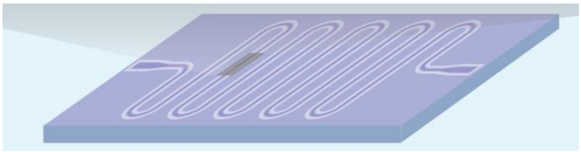
Z=200 Ω

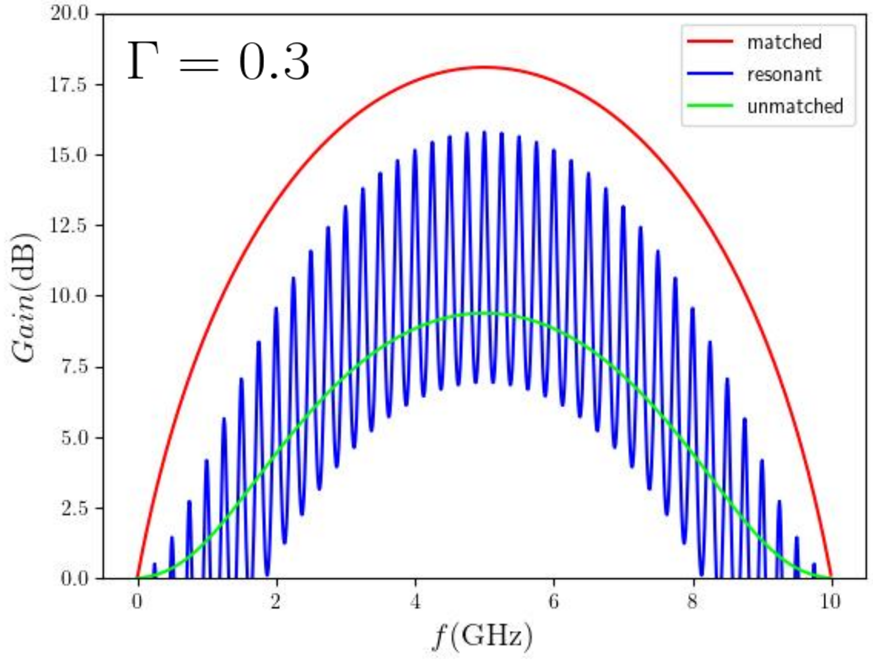
$$\Gamma = \frac{Z - Z_L}{Z + Z_L}$$

$$t_n = \frac{1}{1 - \Gamma^2 e^{i2k_j L}}$$

$$BW = \frac{\sqrt{1 - \Gamma^2}}{\Gamma} \frac{v}{4\pi l}$$

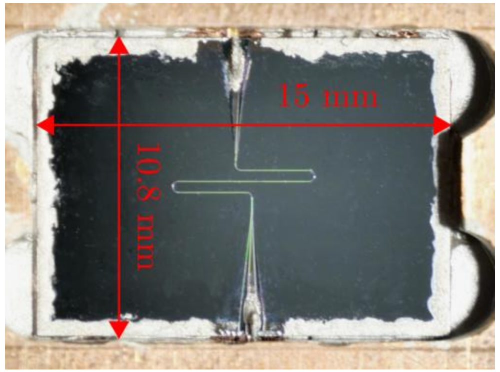


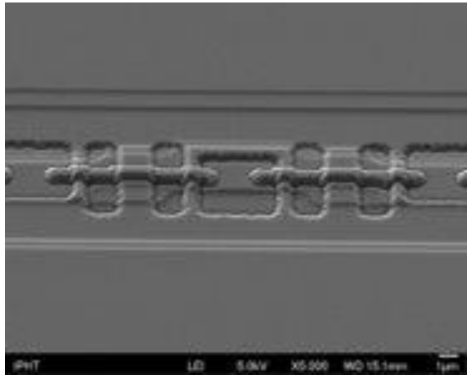


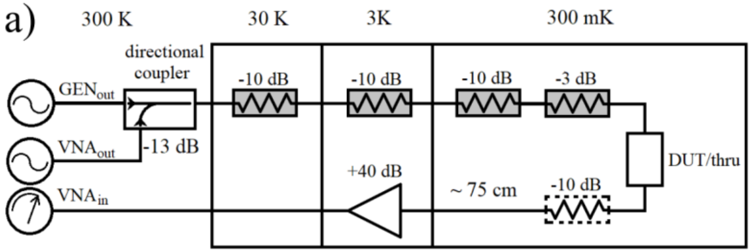


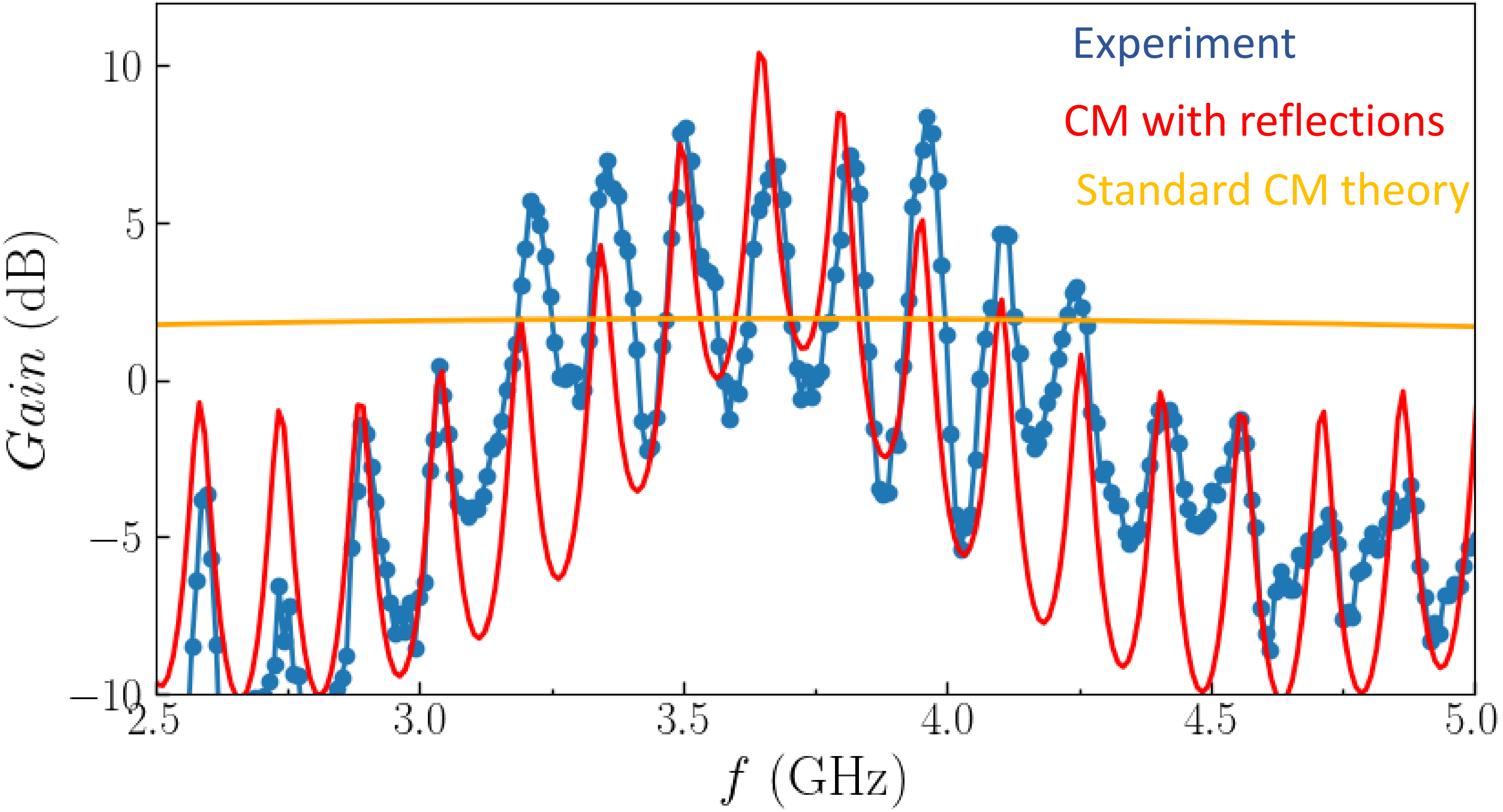
# Nb JJ TWPA - experiment

- Array of 2000 JJ in coplanar waveguide, l=11mm, $v_{ph}$ = 0.14c, tested at T = 3.5K



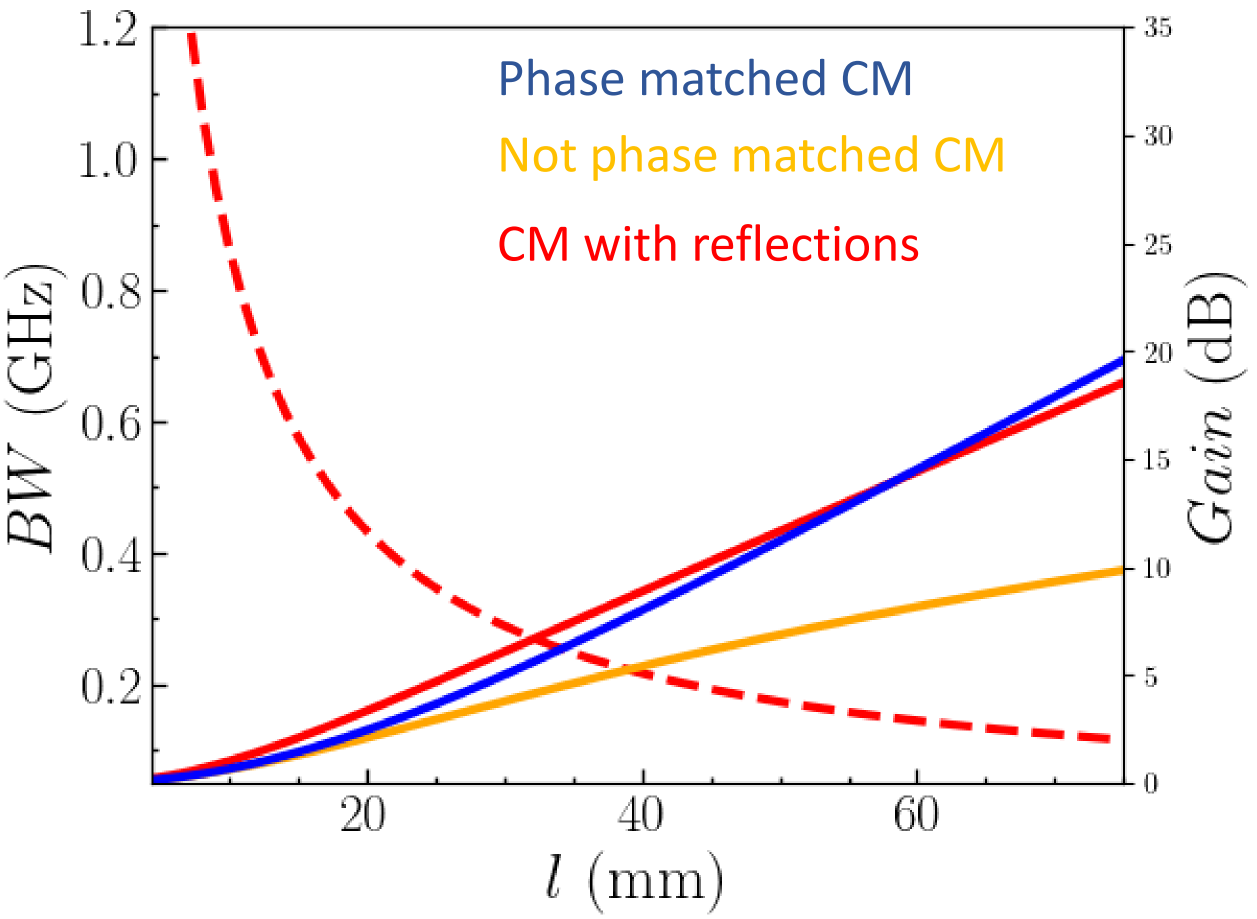






# Conclusion

- Modified CM theory
  – unmatched TWPA with finite length
- Proper understanding of gain ripples and amplifier bandwidth
- Can be utilized in TWPA design
- The transition regime between traveling-wave and resonant parametric amplifier
- Gain 10dB at 11 mm length without phase matching BW~ 100MHz, – tunable design



Thank you for your attention!

S. Kern, et al. arXiv:2203.02448 (2022)

QUANTERA
AGENTÚRA NA PODPORU VÝSKUMU A VÝVOJA
NATO OTAN