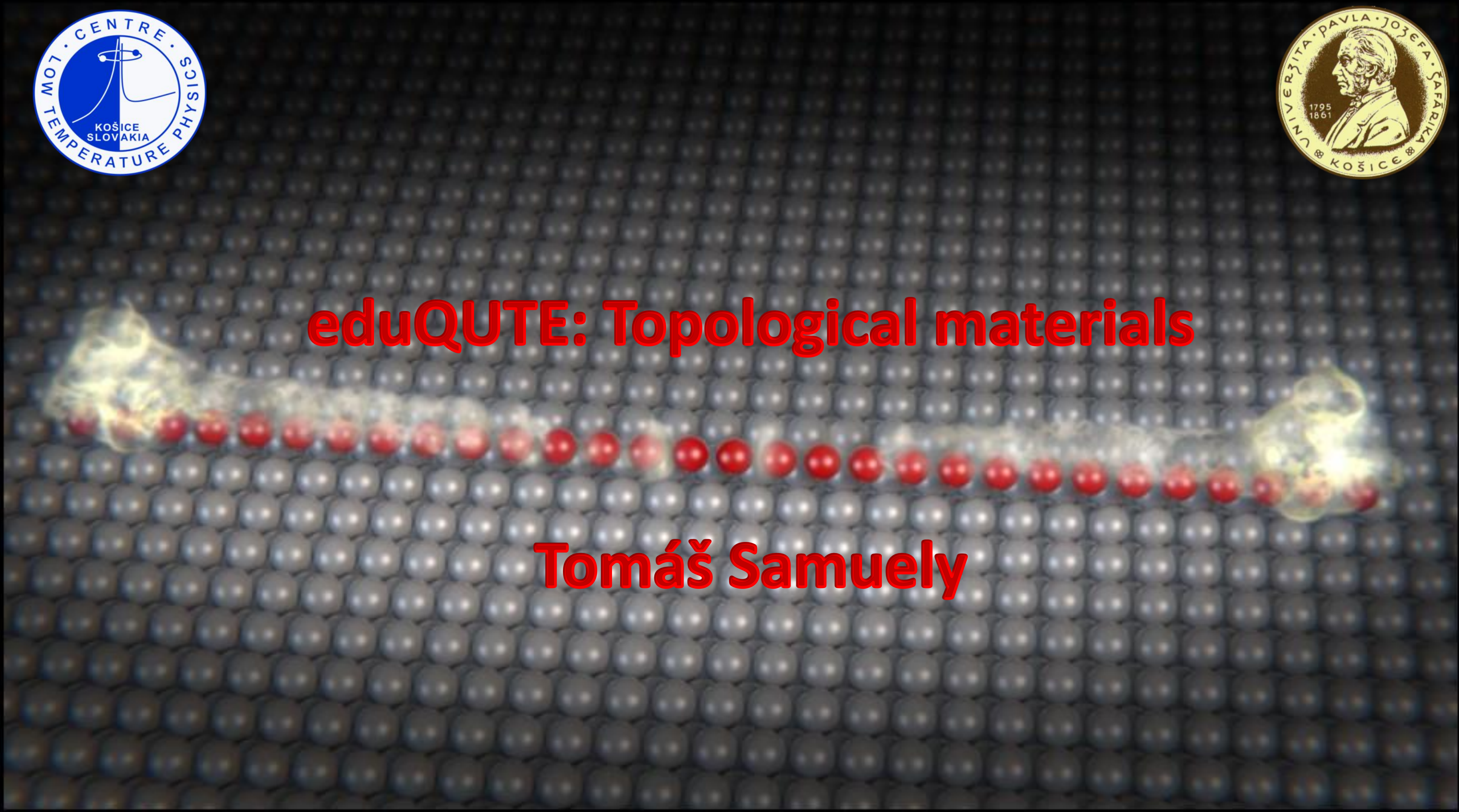


eduQUTE: Topological materials

Tomáš Samuely

Errors

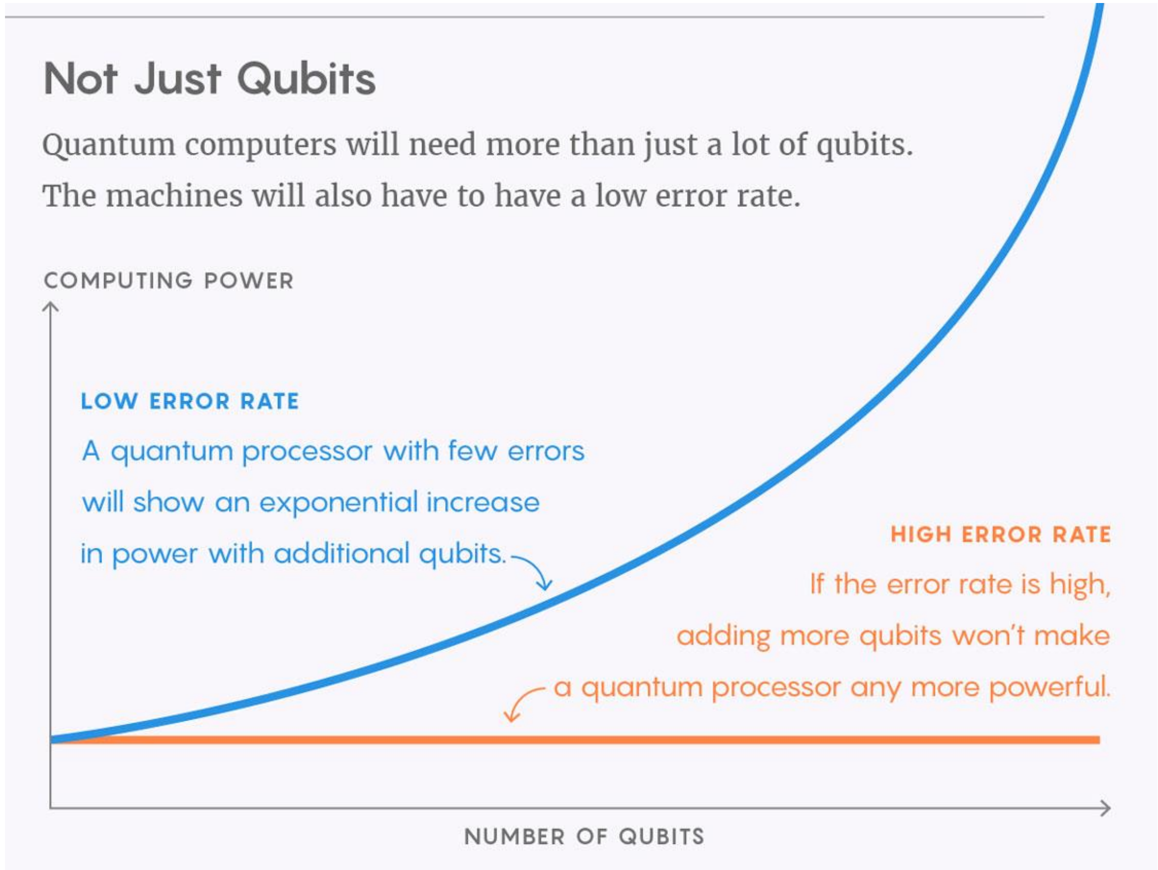
To construct a genuine “logical qubit” on which computation with **error correction** can be performed, you need many physical qubits.

How many?

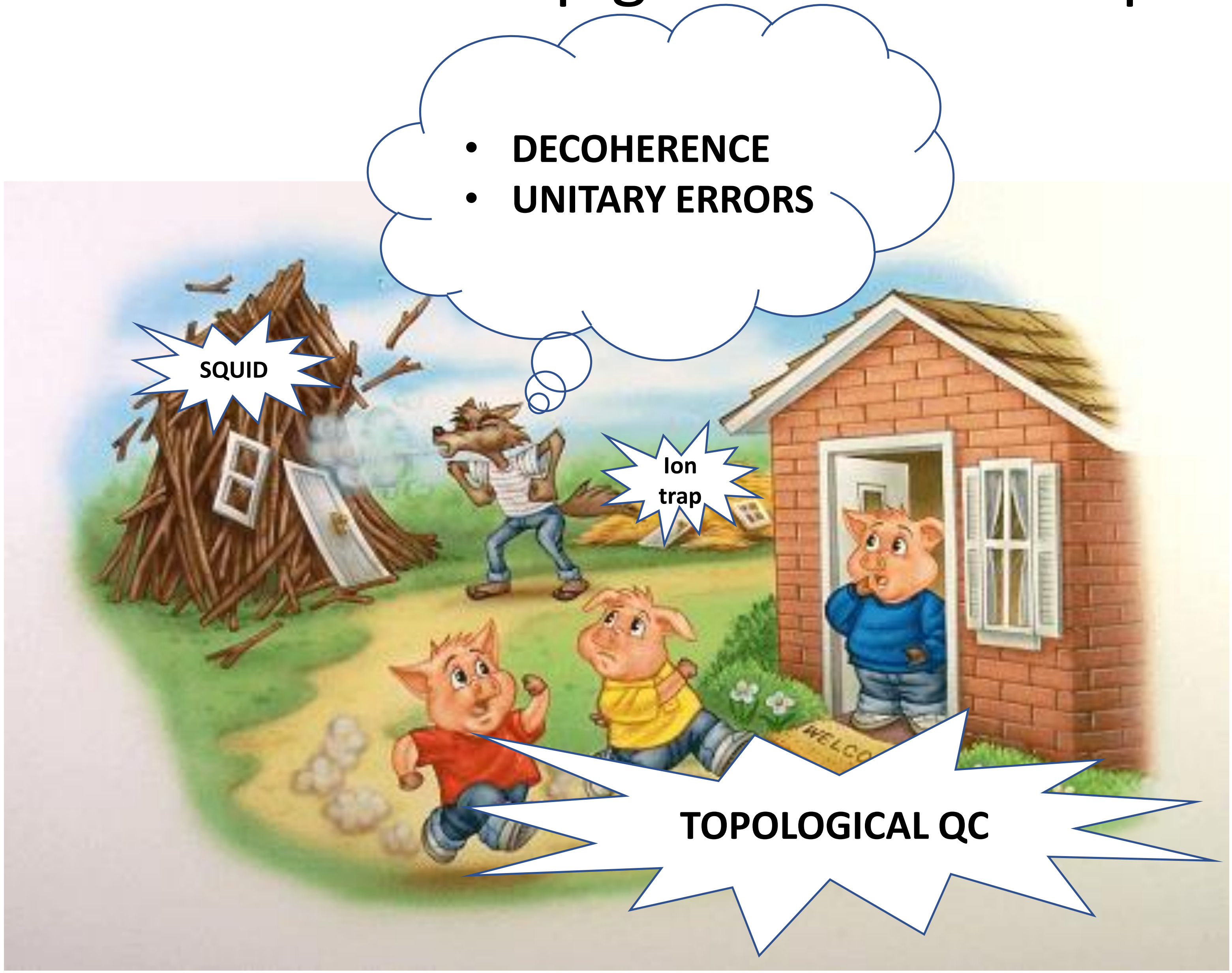
Alán Aspuru-Guzik of Harvard University estimates that around 10,000. If the qubits get much better, he said, this number could come down to a few thousand or even hundreds.

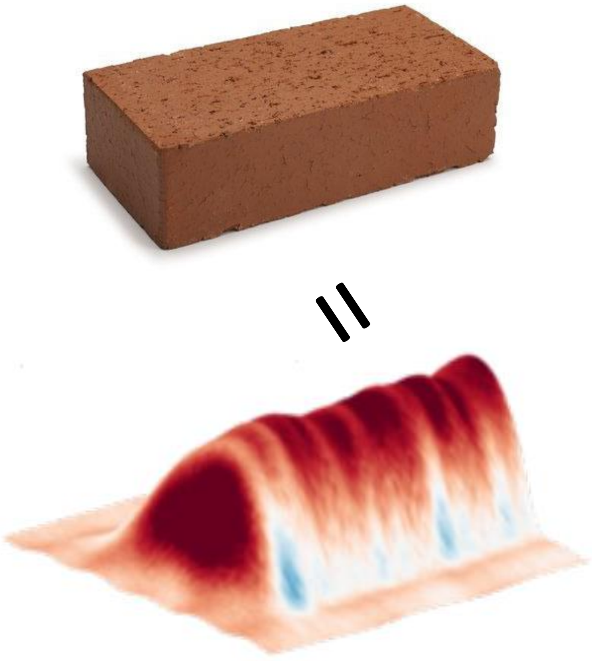
Jens Eisert of the Free University of Berlin is less pessimistic, saying that on the order of 800 physical qubits might already be enough.

Gil Kalai of the Hebrew University of Jerusalem in Israel: I think that the effort required to obtain a low enough error level for any implementation of universal quantum circuits increases exponentially with the number of qubits, and thus, **quantum computers are not possible**.



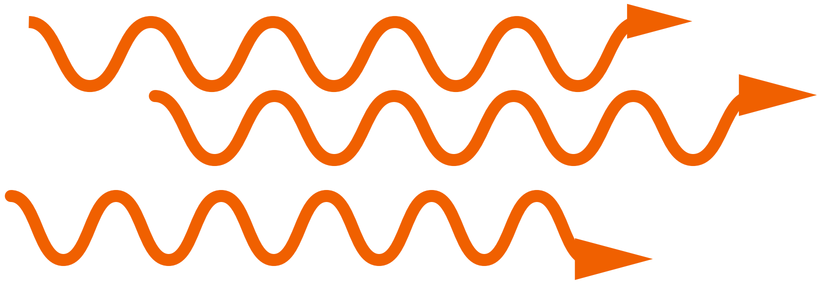
Three little pigs build ~~houses~~ quantum computers



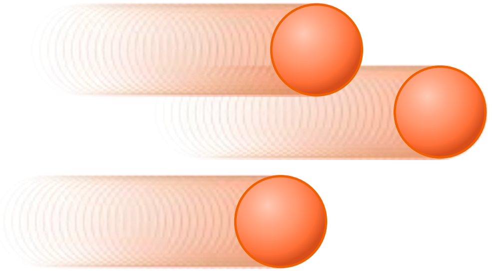


topologically protected qubit

Before quantum theory



Waves

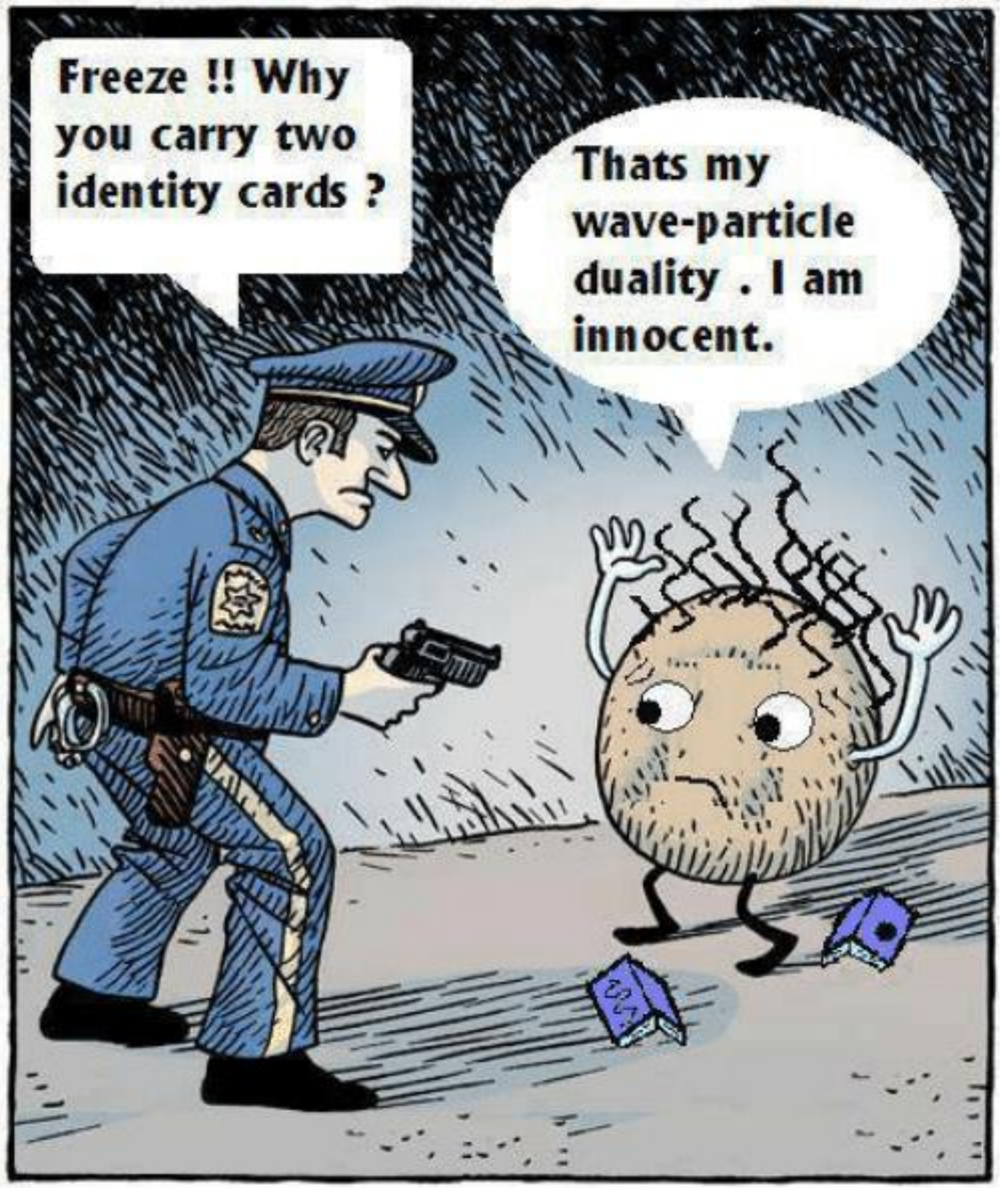


Particles

James Clerk Maxwell

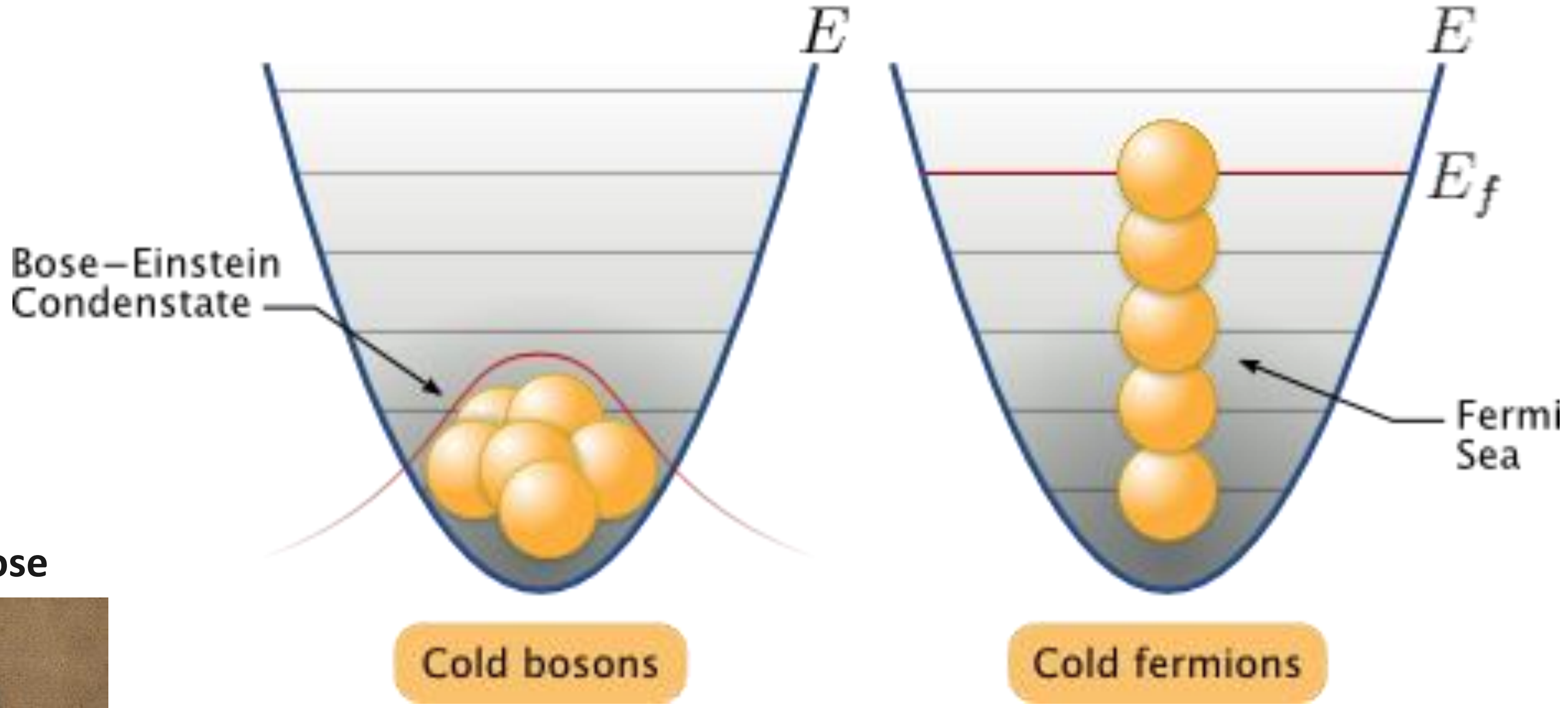
Isaac Newton

Quantum theory replaced dualistic view with a unified one



A “quarticle” cannot be associated with a definite location in space. Instead, the possible results of measuring its position are given by a probability distribution. And that distribution is given as the square of a space-filling field, its so-called wave function.

For collections of identical quarticles, a new dualism appears.



Satyendra Bose

Enrico Fermi

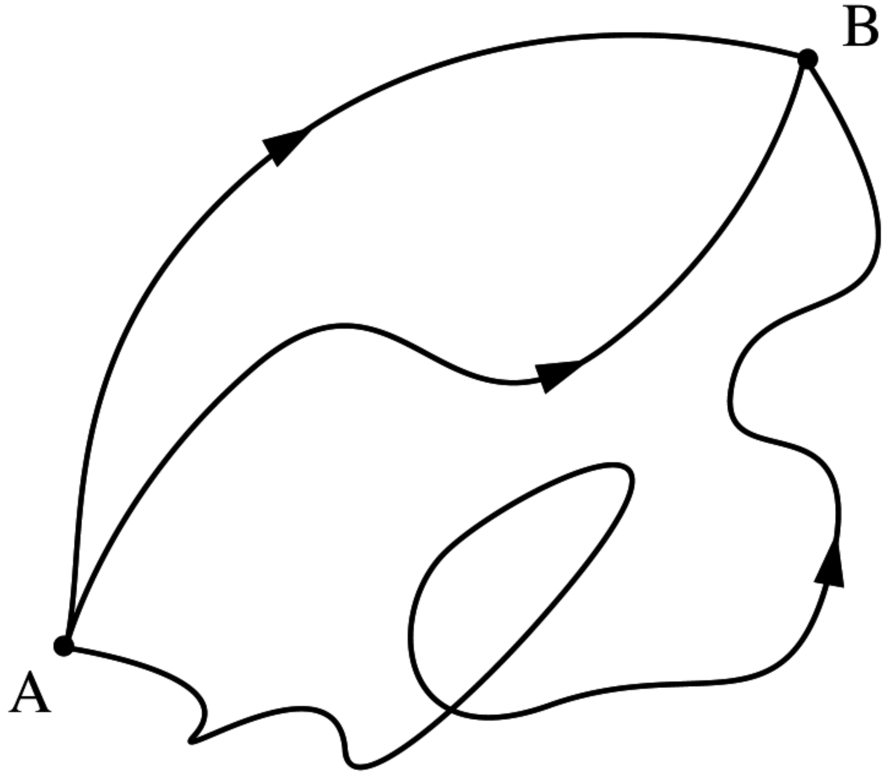
quantum statistics

Quantum statistics reflects the topology of quarticle trajectories:

Probability for a process is the square of the amplitude of its wave function.

$\Psi(x,t)$ = single-valued probability amplitude at (x,t)

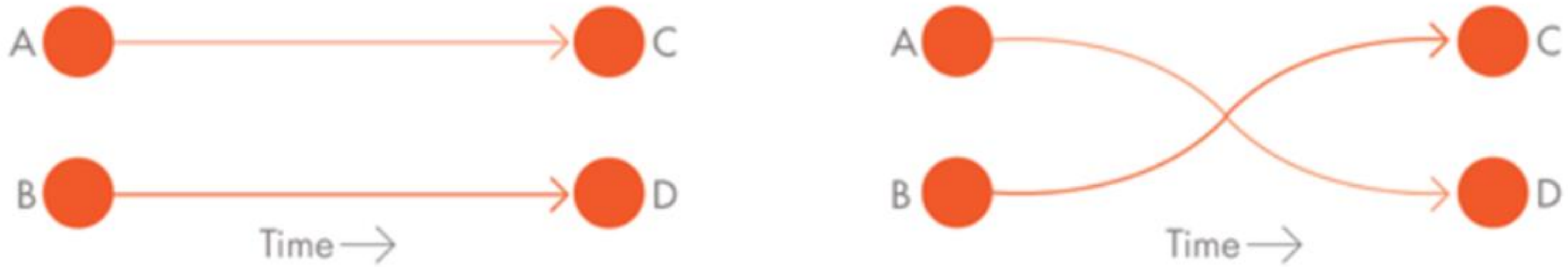
$\Psi^*(x,t)\Psi(x,t)$ = probability of finding particle at x at time t provided the wavefunction is normalized.



Path integral – a functional integral over an infinity of quantum-mechanically possible trajectories allows one to compute a quantum amplitude for a quarticle moving from point A at some time t_0 to point B at some other time t_1.

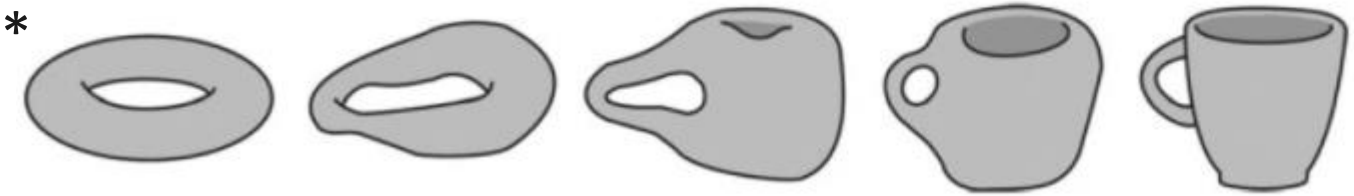
Quantum statistics reflects the topology of quarticle trajectories:

In calculating the total amplitude for two indistinguishable quarticles that begin at positions A and B and end at positions C and D, we must take into account contributions from every possible motion connecting the starting positions to the end points.



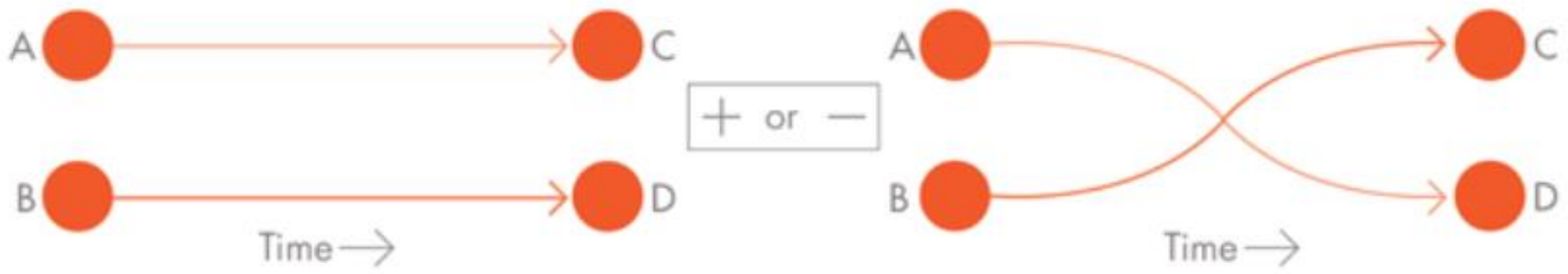
Two classes of trajectories – same result (quarticles are indistinguishable), different topology!*

*



Quantum statistics reflects the topology of quarticle trajectories:

There are two mathematically consistent possibilities, to combine the contributions from those two classes. We can add them, or we can subtract them.

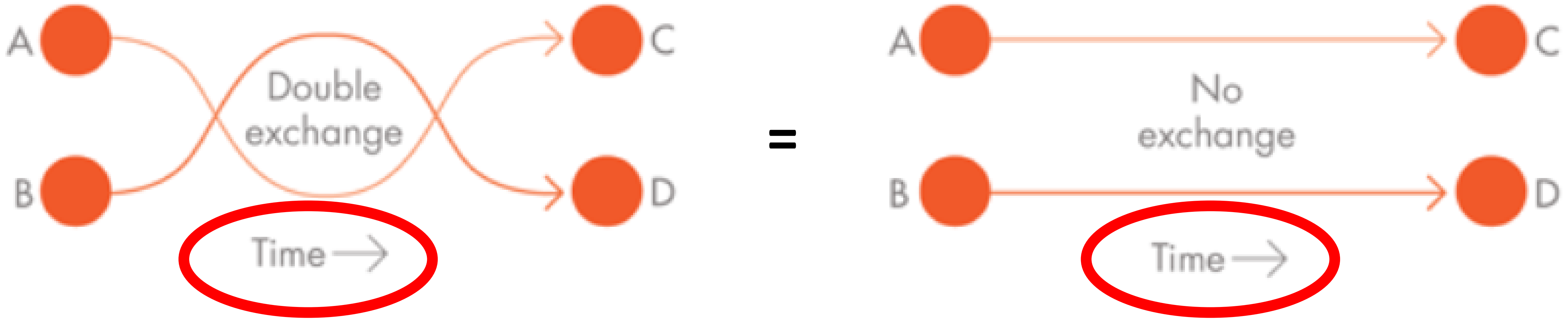


The **+** option gives us bosons, while the **-** option gives us fermions. All the characteristic properties of bosons and fermions can be deduced from those basic rules.

Indistinguishability and the topology of motion in space-time determines these basic properties of matter .

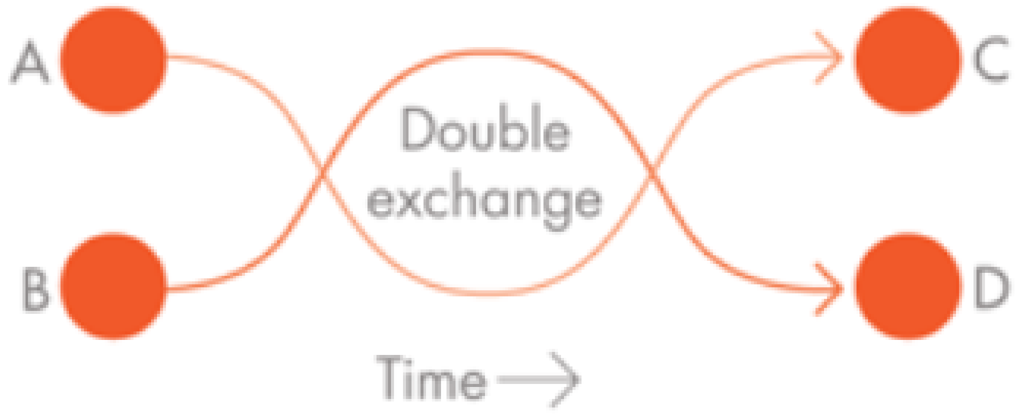
Knot or not?

In 4D all knots can be unraveled completely!



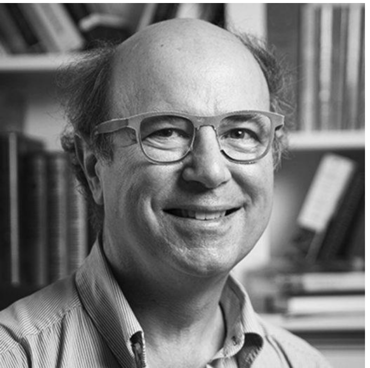
Knot or not?

2D space = 3D spacetime



≠

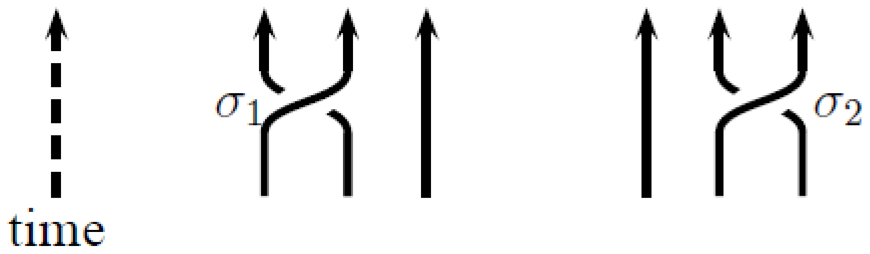
Frank Wilczek

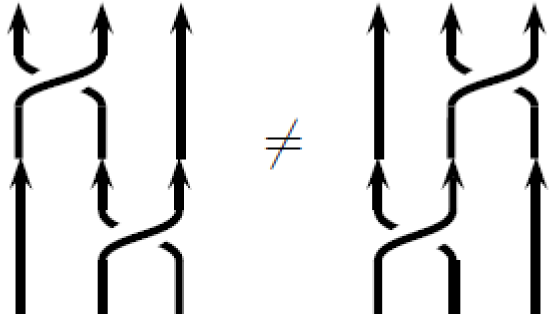


Richer topology ⇒ more possibilities for quantum statistics ⇒ more than just bosons and fermions ⇒ **ANYONS** with memory!
Value of the amplitude provides a record of their relative motion in spacetime.

Anyons

Braiding





Non-Abelian: $\sigma_2\sigma_1 \neq \sigma_1\sigma_2$

If two anyons are exchanged counterclockwise,
the wavefunction can change by an arbitrary phase:

$\psi(r1, r2) \rightarrow e^{i\theta}\psi(r1, r2)$
(Bosons: $\theta = 0$, Fermions: $\theta = \pi$)

A second counter-clockwise exchange need not lead back to the initial state but can result in a non-trivial phase:

$\psi(r1, r2) \rightarrow e^{2i\theta}\psi(r1, r2)$

If anyons are non-Abelian*, braiding will cause non-trivial rotations within the Hilbert space. Systems of many non-Abelian anyons build up a gigantic collective memory, which can serve as a platform for **topological quantum computing.**

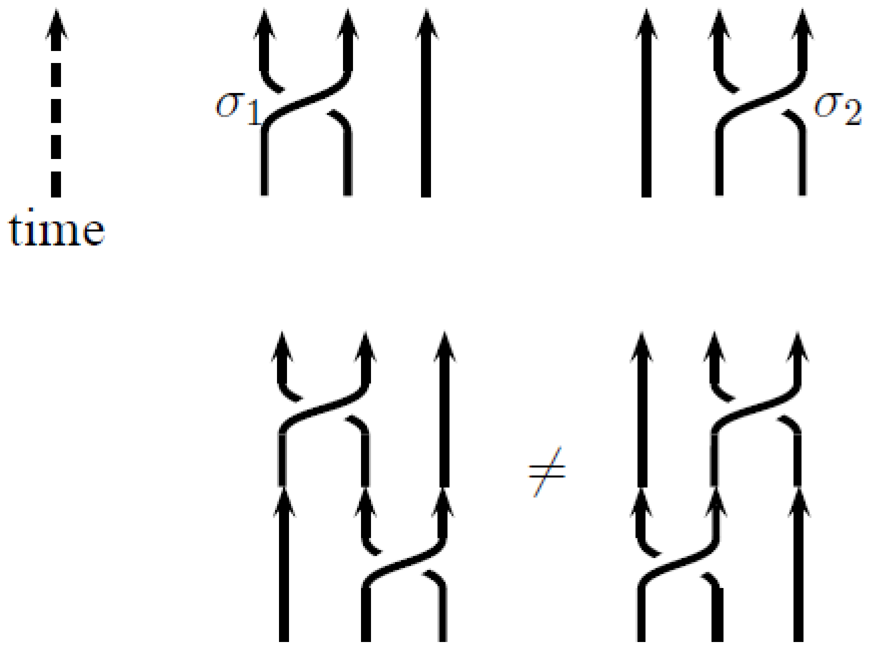
*e.g. fractional quantum Hall effect – Abelian anyons

Anyons

Protected from:

- decoherence, because they are non-local quasiparticles. Their subspace of degenerate ground states, used for computation, is separated from the rest of the spectrum by an energy gap.
- unitary errors, since the unitary transformations associated with braiding quasiparticles are sensitive only to the topology of the quasiparticle trajectories, and not to their geometry or dynamics.

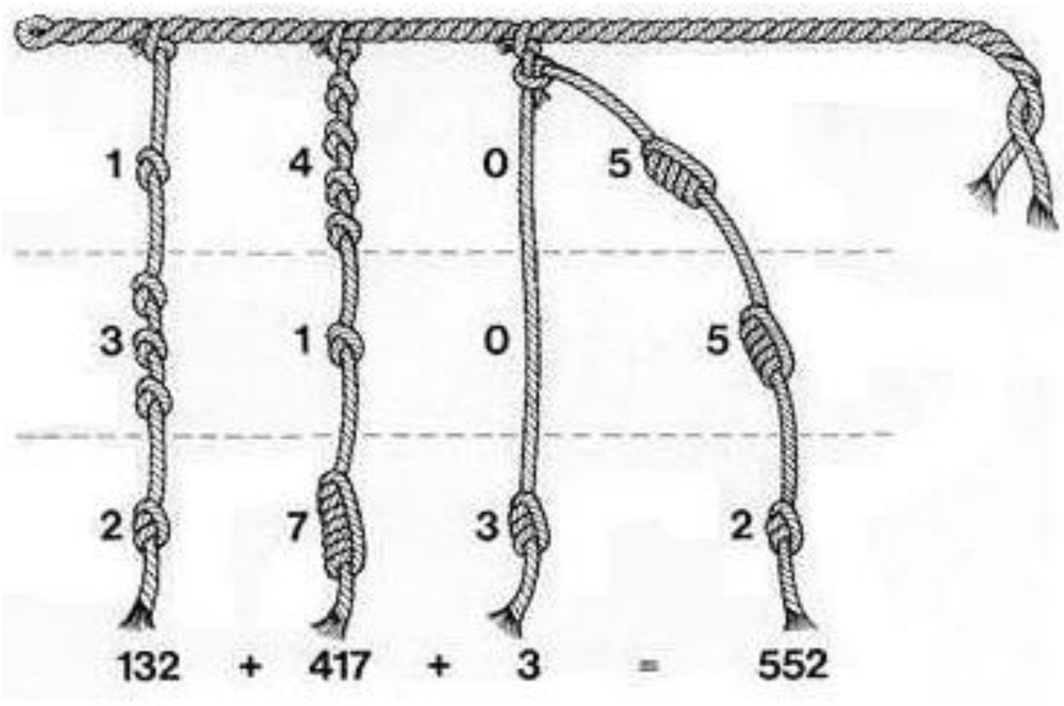
Braiding



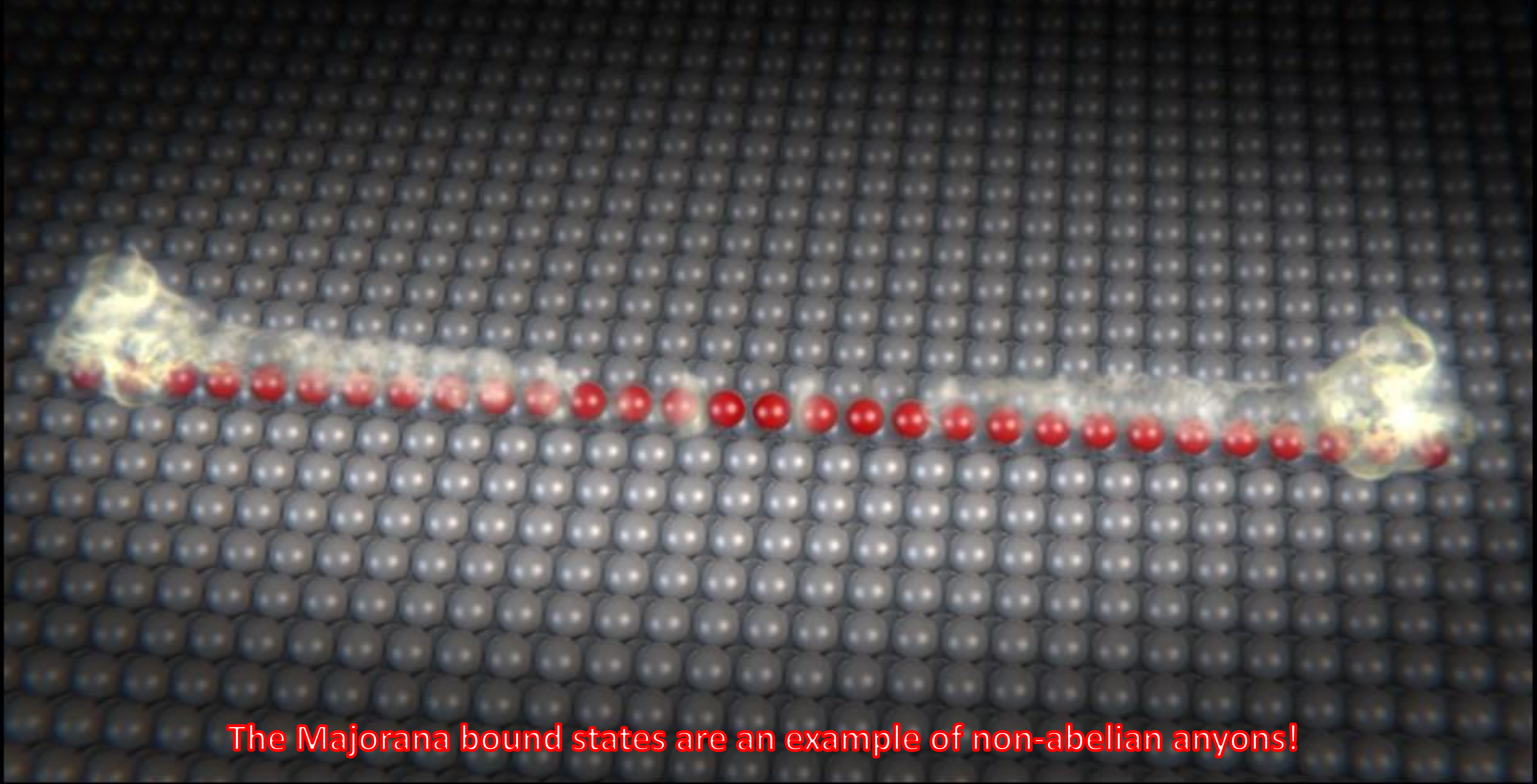
Non-Abelian: $\sigma_2\sigma_1 \neq \sigma_1\sigma_2$

Braiding

Computing with anyons exploits their ability to map their knotted histories into quantum-mechanical amplitudes. Topological quantum computing is therefore a modernization of quipu, the Incan technology for computation and encryption with knots.



Do non-Abelian anyons exist?



The Majorana bound states are an example of non-abelian anyons!

Building Majorana bound states

We are looking for Majorana excitations which are their own antiparticle, i.e., whose creation and annihilation operators satisfy

$$\gamma = \gamma^\dagger$$

Such an operator, which consists in equal parts of electrons and holes, is e.g.:

$$\gamma = c + c^\dagger$$

BCS superconductors have fermionic quasiparticle excitations described by linear combinations of creation and annihilation operators:

$$\gamma = uc + vc^\dagger$$

The prefactors in this linear combination depend on the energy of the (Bogoliubov) quasiparticle excitation. An excitation far above the superconducting gap will behave like an electron and thus u = 1 and v = 0. Far below the superconducting gap will look like a hole; u = 0 and v = 1. A Majorana excitation has equal amplitudes of c and $c^\dagger$, i.e., we are looking for a midgap excitation with u = v.

Building Majorana bound states

But in BCS, we have spins!!!

$$\gamma_\uparrow = uc_\uparrow + vc_\downarrow^\dagger \qquad\qquad \gamma_\uparrow \neq \gamma_\uparrow^\dagger$$

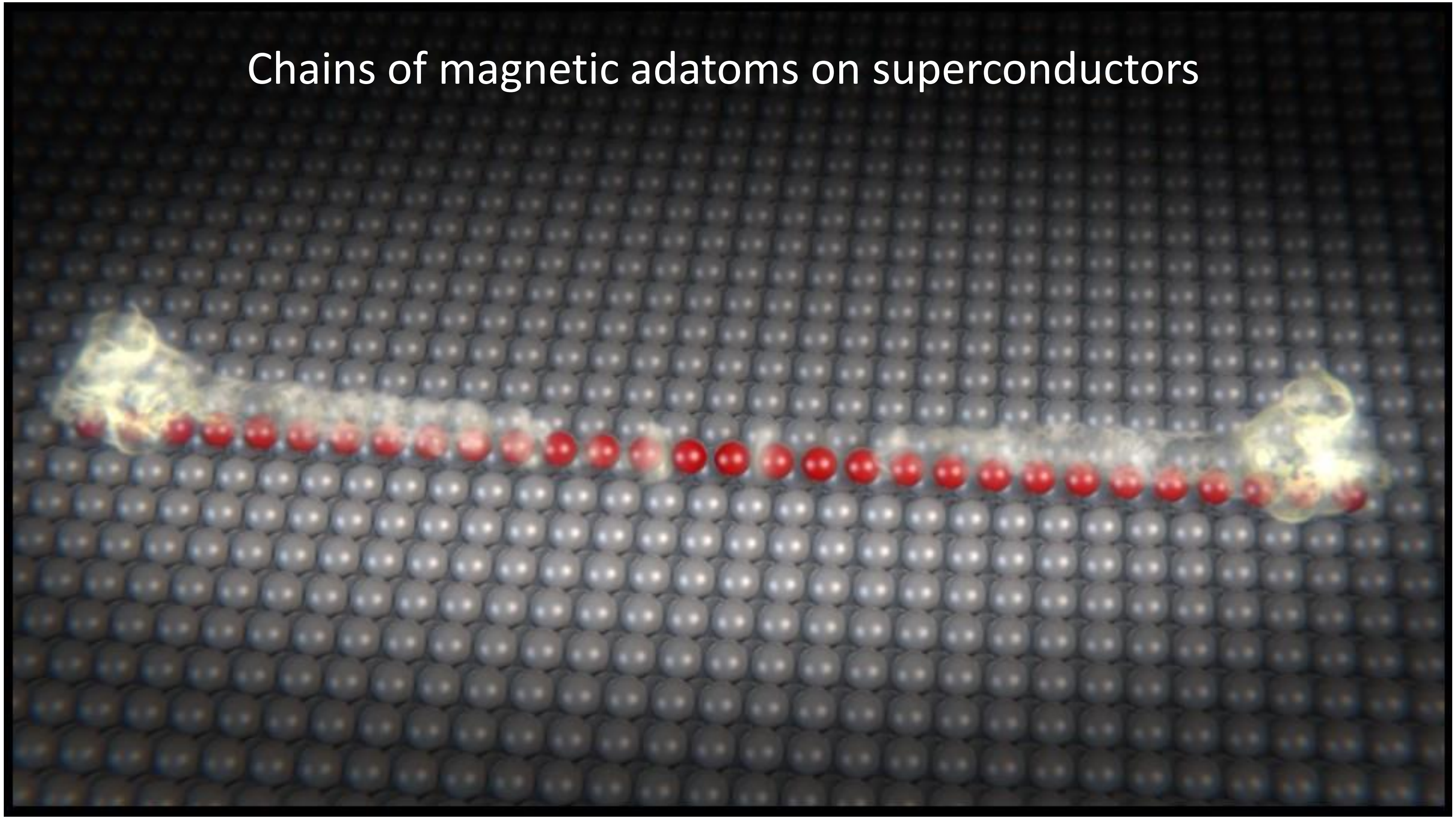
Thus, we should be looking for Majoranas as zero-energy excitations in superconductors made of spinless fermions. Because of the Pauli principle, the Cooper pair wavefunction must be antisymmetric. For spinless fermions, there is no spin part of the Cooper pair wavefunction and the antisymmetry must be in the orbital part. Hence we seek zero-energy excitations in spinless **p-wave superconductors**!

Experimentally accessible systems must have

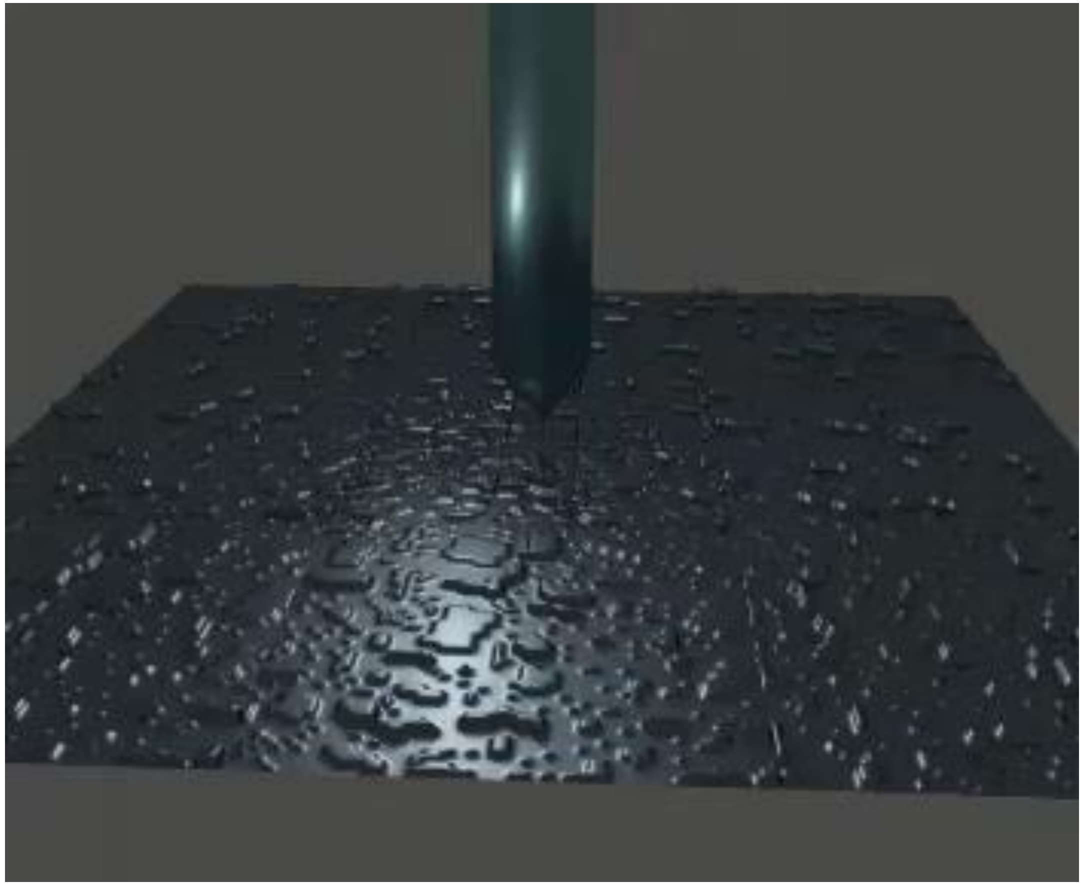
- proximity coupling to a conventional s-wave superconductor
- spin polarization
- spin-orbit coupling

Building Majorana bound states

E.g. we can put a one-dimensional spin polarized system in proximity to a superconductor with spin-orbit coupling. This one has a small p-wave admixture to the s-wave pairing. Unlike the s-wave correlations, the p-wave correlations can transfer to the spin-polarized system.



Scanning Tunneling Microscopy

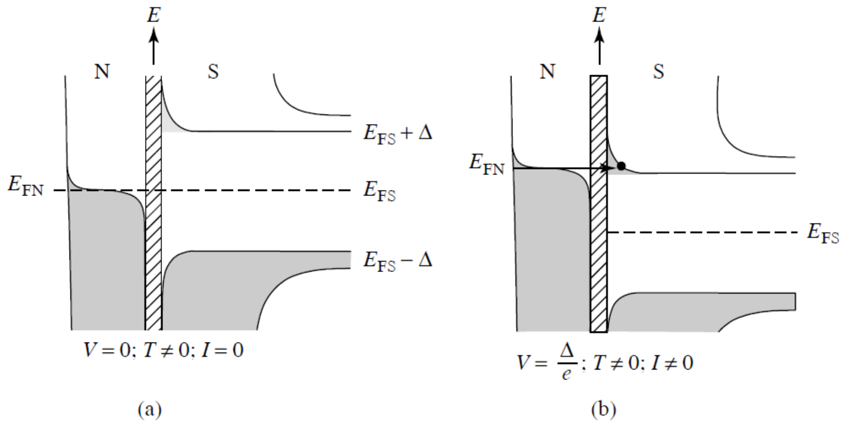


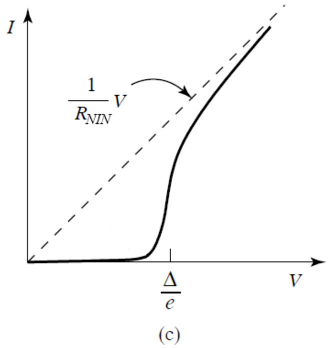
Tunneling spectroscopy

$$\frac{dI_{NIS}}{dV} = G_{NIN} \int_{-\infty}^{\infty} \frac{\rho_S(E)}{\rho_N(E_F)} \left[\frac{-\partial f(E+eV)}{\partial (eV)} \right] dE$$

$k_B T << eV$:

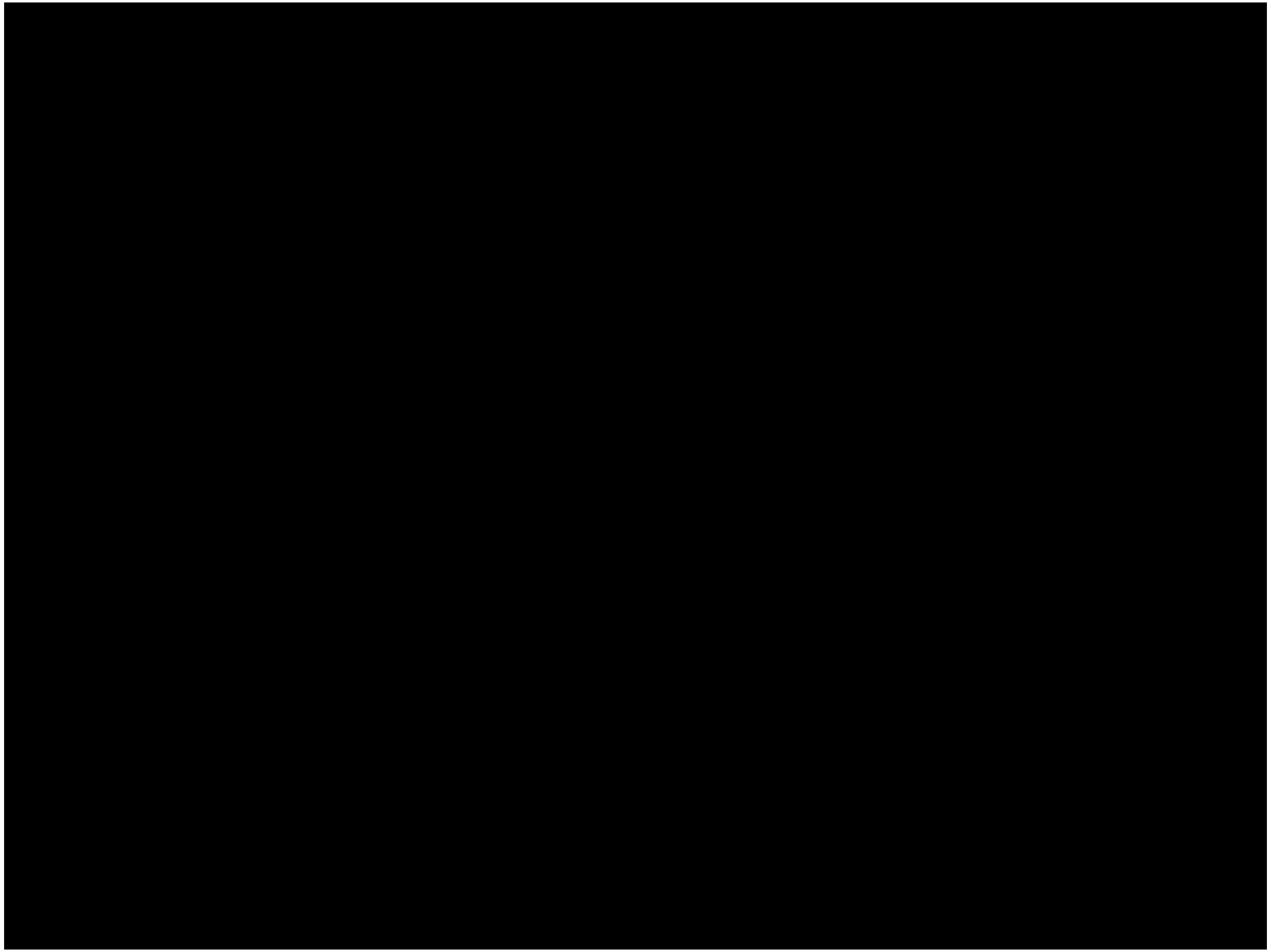
$$\frac{dI_{NIS}}{dV} = G_{NIN} \frac{eV}{[(eV)^2 - \Delta^2]^{\frac{1}{2}}} = G_{NIN} \rho_S(E = eV)$$





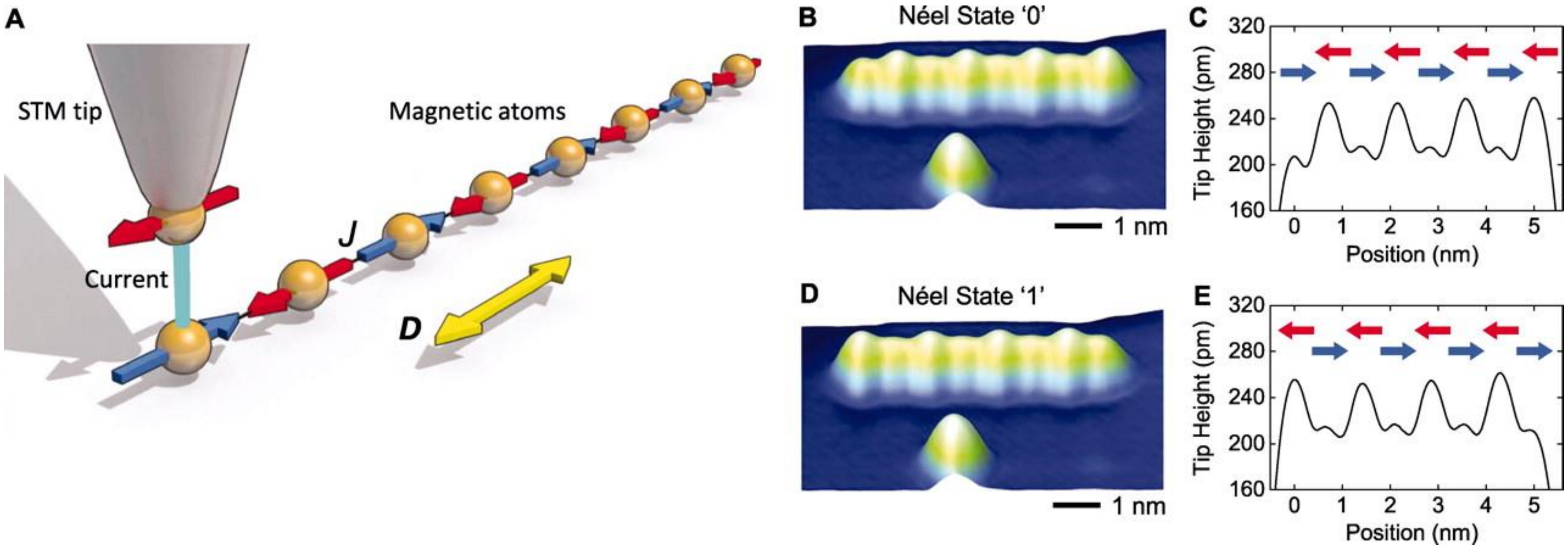
$$\frac{\rho_S(E)}{\rho_N(0)} = \begin{cases} \frac{E}{(E^2 - \Delta^2)^{1/2}}; & |E| \geq \Delta \\ 0; & |E| < \Delta \end{cases}$$

Nanomanipulation



CO oc Cu(111)

Spin polarized STM

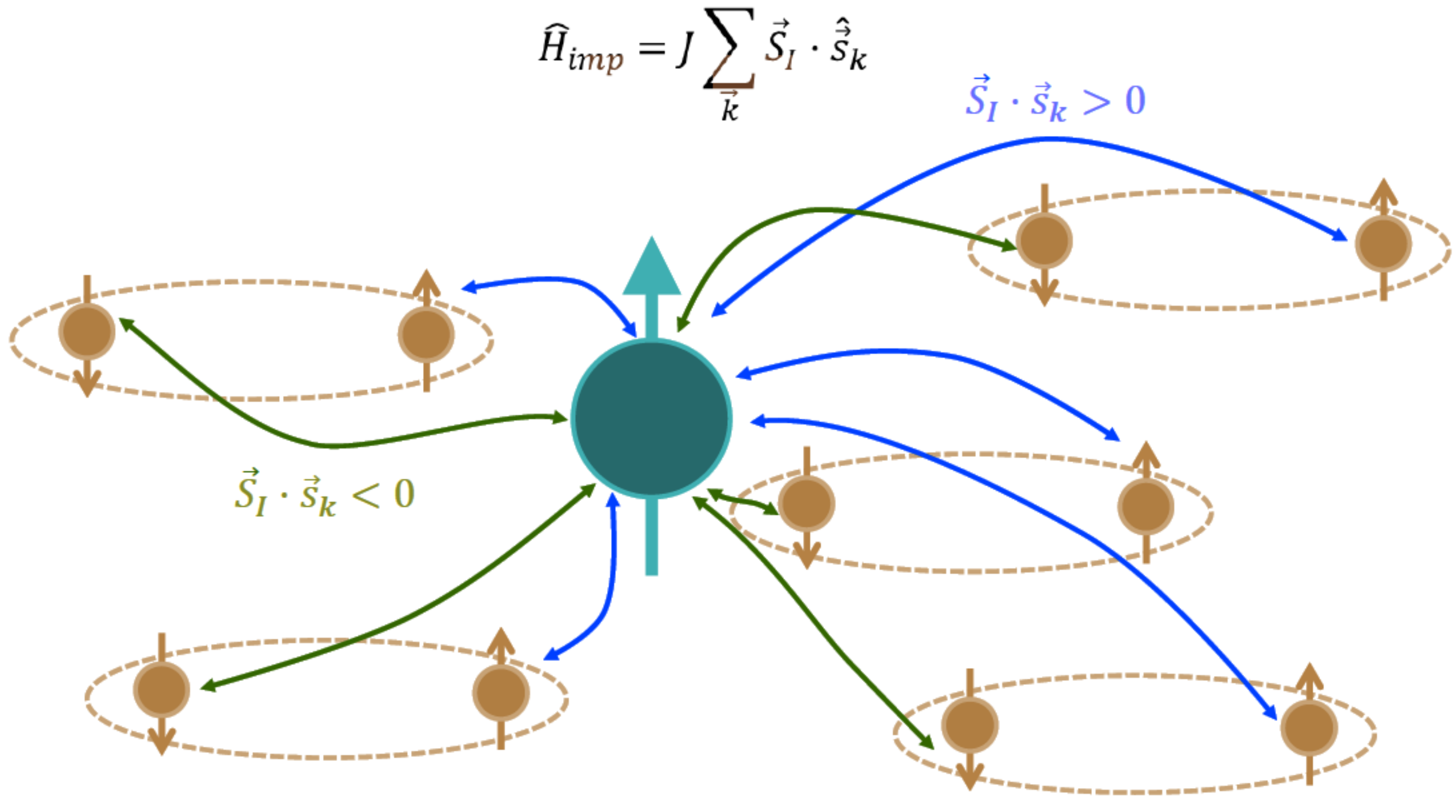


The Fe atoms were placed at a binding site on a Cu_2N surface, for which Fe has a large magnetic anisotropy field that aligns its spin to the resulting easy axis *D.*

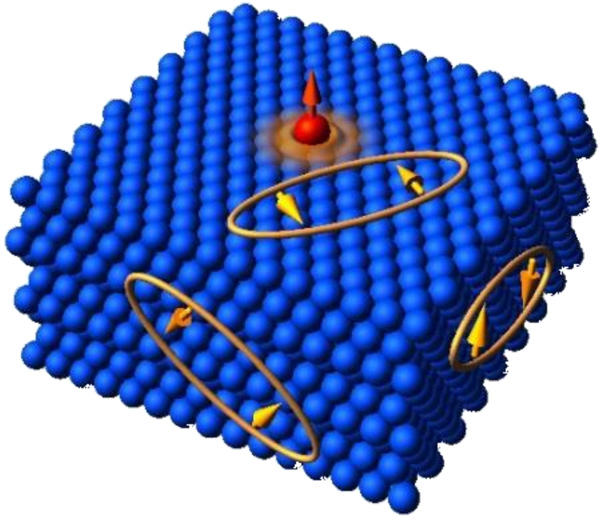
Néel states are stable at voltage < 2 mV (current 1 pA).

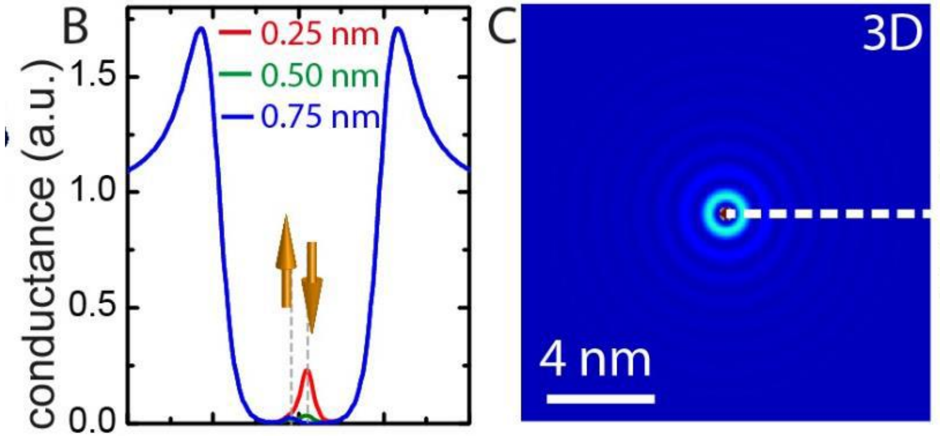
Yu-Shiba-Rusinov states

Classical magnetic impurities in a superconductor
Classical impurity approximation: the impurity behaves as a local magnetic field



Yu-Shiba-Rusinov states



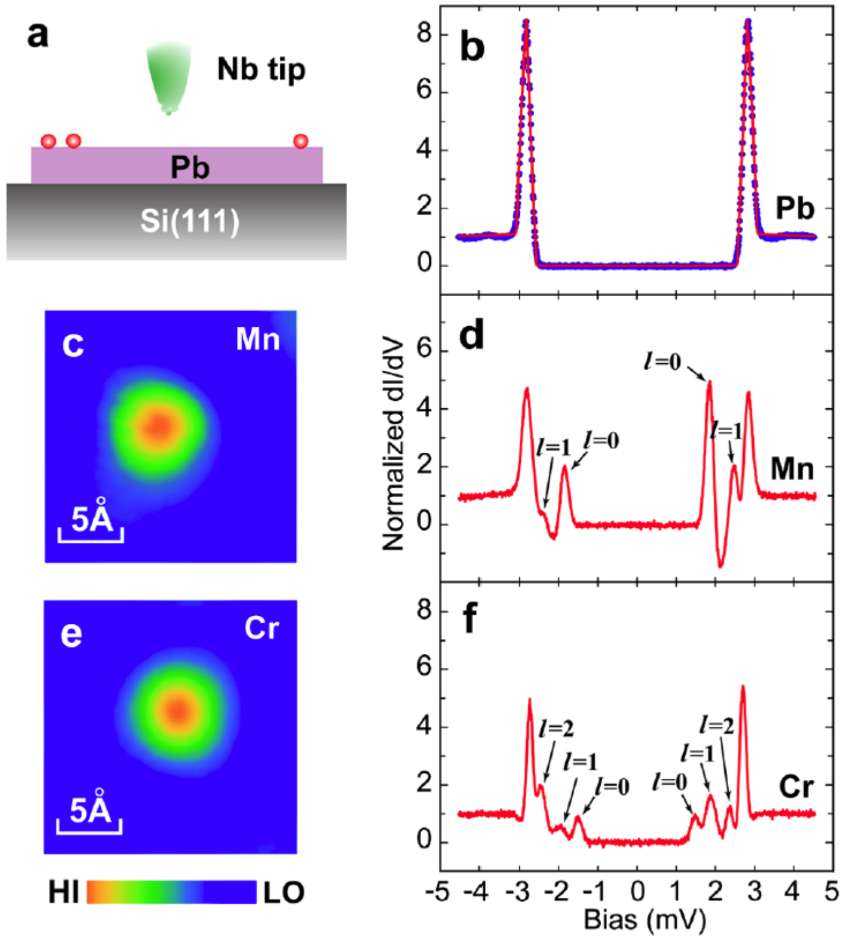


Yu-Shiba-Rusinov states

The number of Shiba peaks depends on the atom nature:

$Mn \rightarrow l$=0,1
$Cr \rightarrow l$=0,1,2

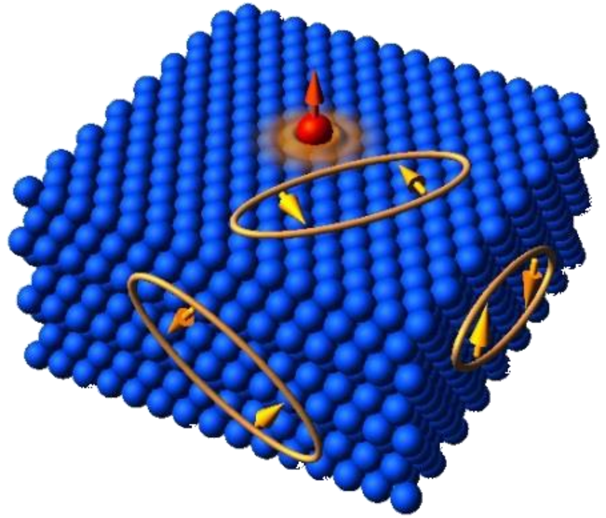
Every peak corresponds to a different diffusion channel for the superconducting electrons.
Extremely local effect of the impurities (a few Å)



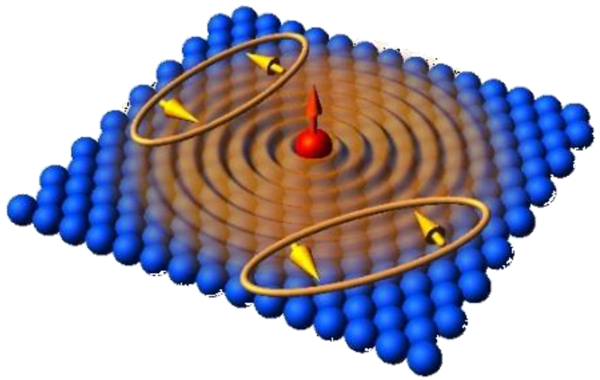
Shuai-Hua Ji et al. *PRL* **100**, 226801 (2008)

Yu-Shiba-Rusinov states

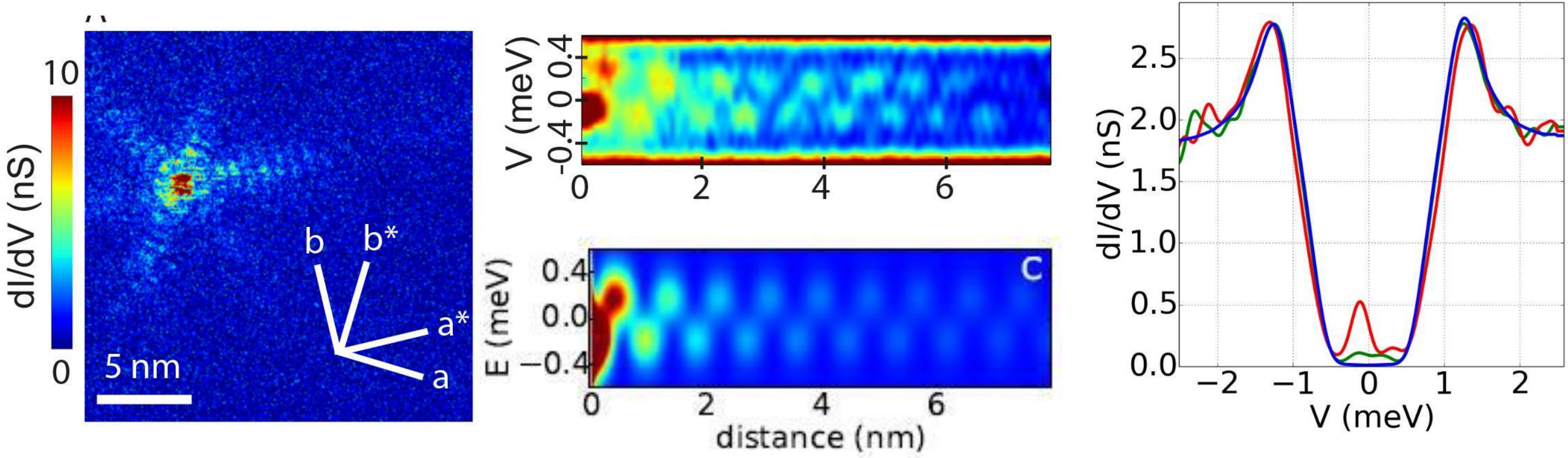
$$\psi_{\pm}^{3D}(r) = \frac{1}{\sqrt{N}} \frac{\sin(k_F r + \delta^{\pm})}{k_F r} e^{-\Delta \sin(\delta^{+} - \delta^{-}) r / \hbar v_F}$$



$$\psi_{\pm}^{2D}(r) = \frac{1}{\sqrt{N}} \frac{\sin\left(k_F r + \delta^{\pm} - \frac{\pi}{4}\right)}{\sqrt{k_F r}} e^{-\Delta \sin(\delta^{+} - \delta^{-}) r / \hbar v_F}$$



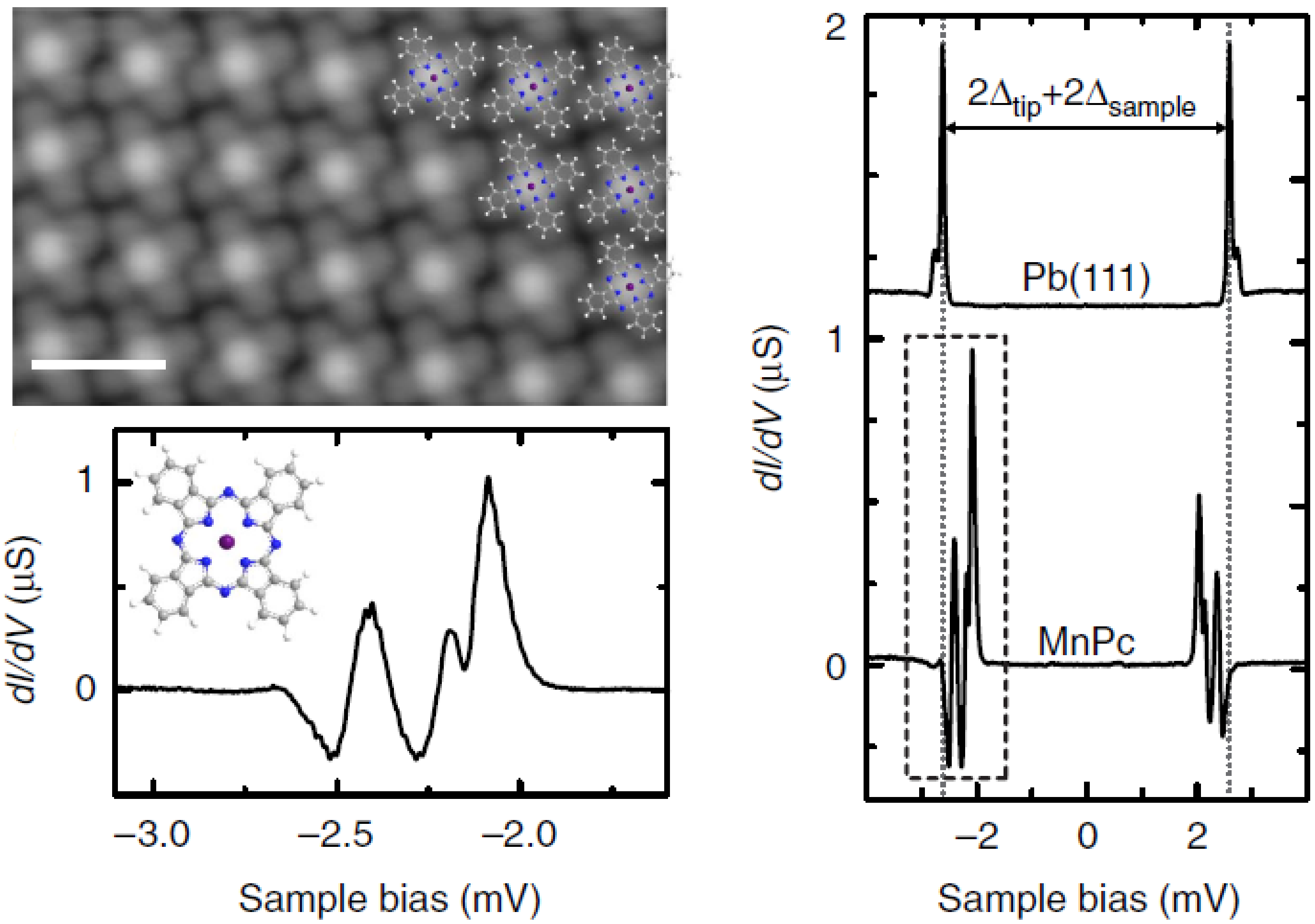
Yu-Shiba-Rusinov states



Electron- and hole-like excitations restore symmetry away from impurity. Good agreement with theoretical calculations for 2D case in the asymptotic limit.

G. Ménard et al. arXiv:1506.06666, Nature Physics(2015)

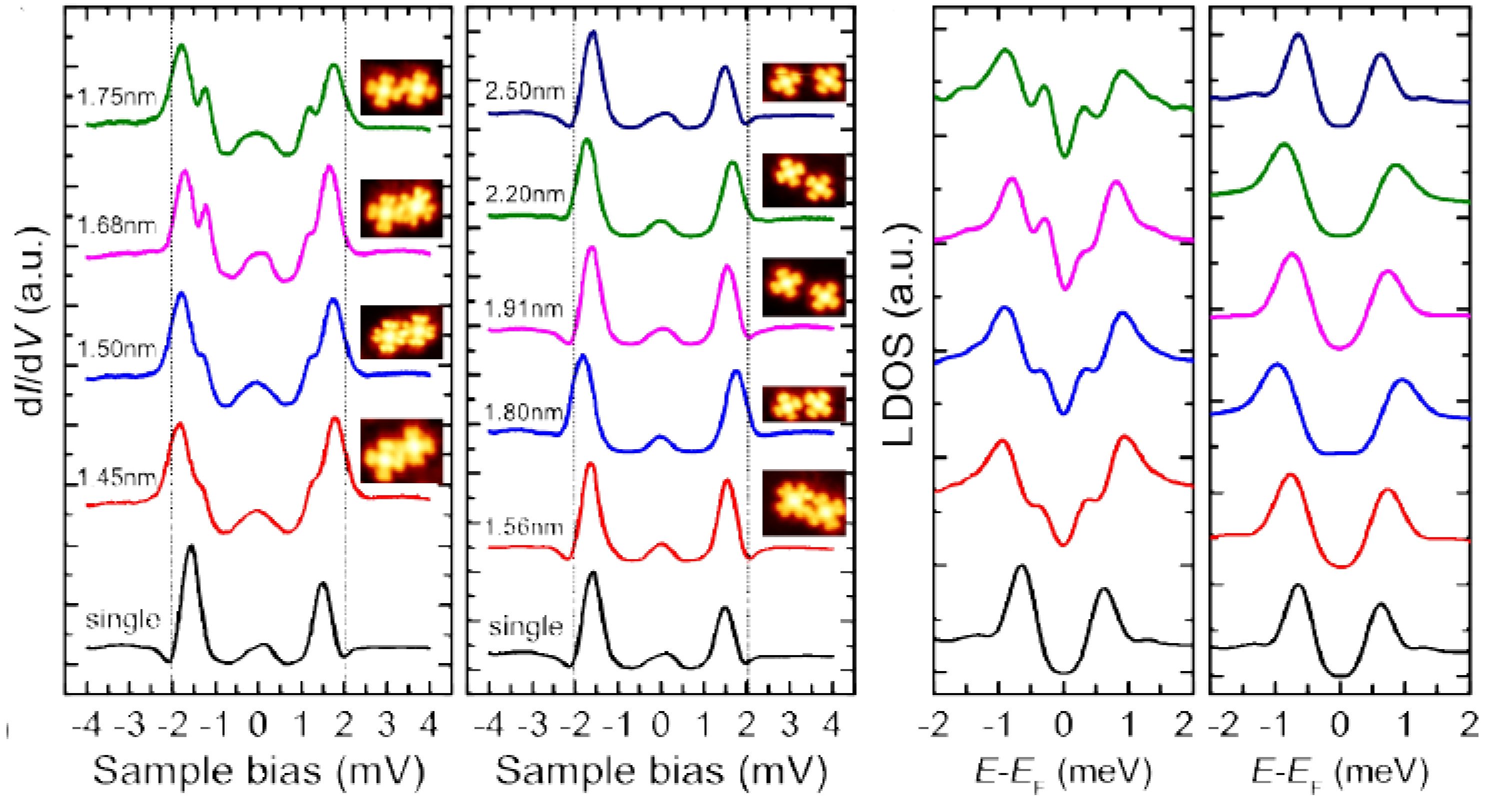
Yu-Shiba-Rusinov states



MnPc adsorption and magnetic fingerprint on Pb(111).

Hatter et al. , Nature Communications 2015

Yu-Shiba-Rusinov states

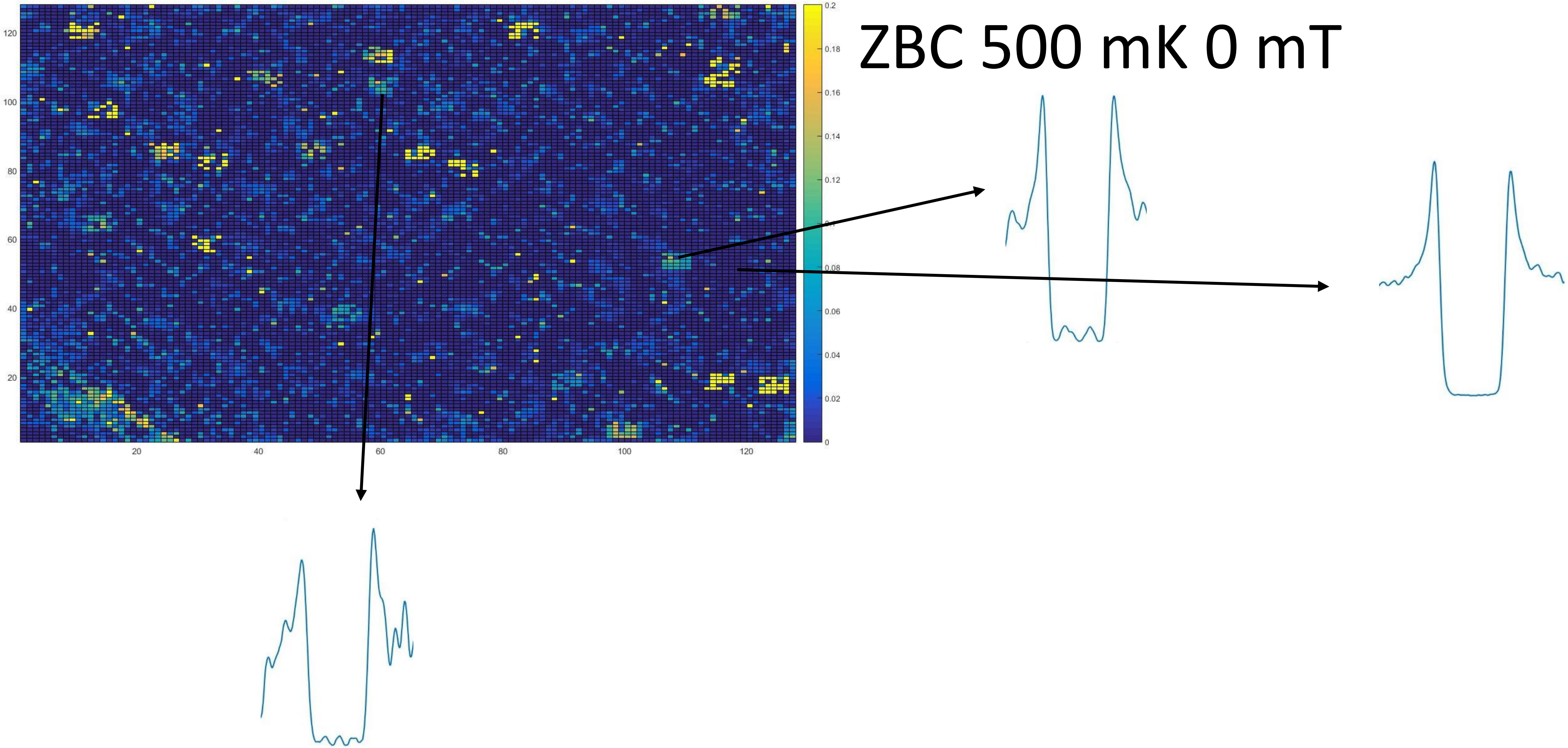


Formation of coupled YSR states on CoPc dimers on $NbSe_2$.

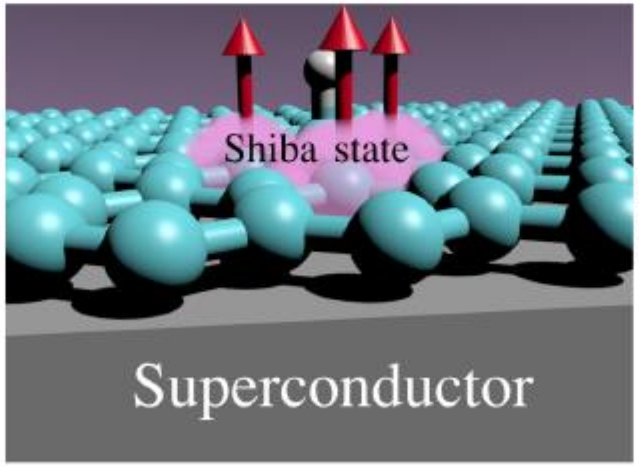
S. Kezilebieke et al. arXiv:1701.03288

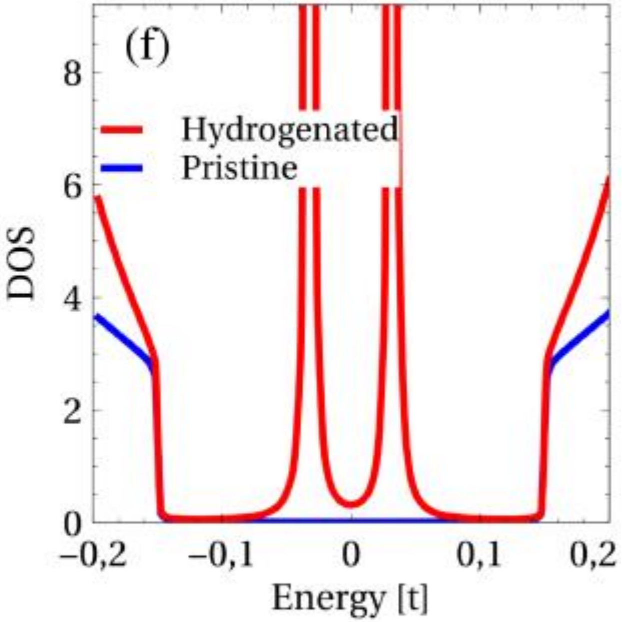
Yu-Shiba-Rusinov states

ZBC 500 mK 0 mT



Yu-Shiba-Rusinov states

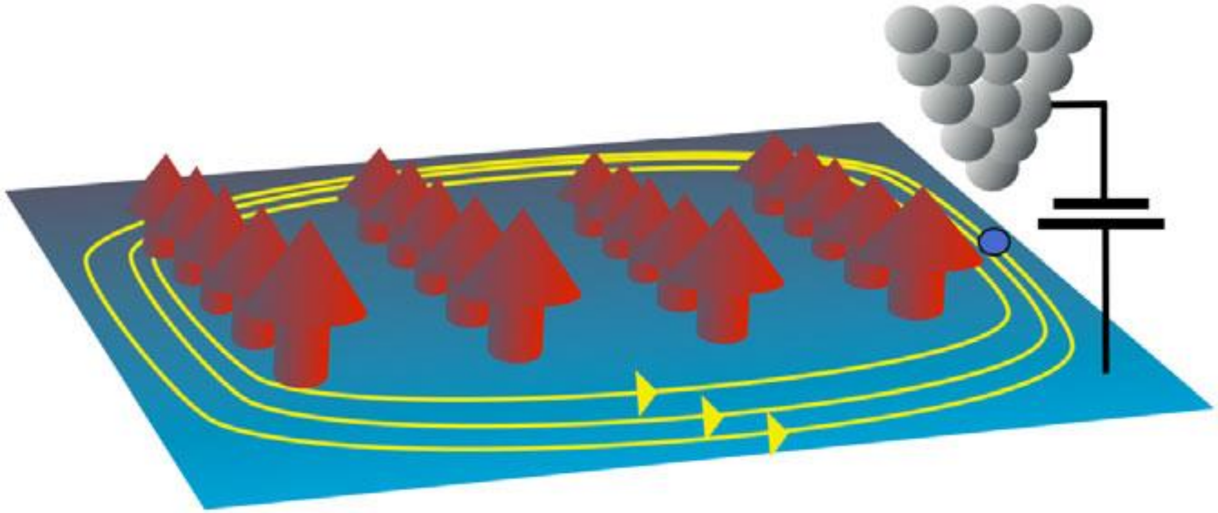




Calculations for hydrogenated graphene in proximity to a superconductor show that individual adatoms induce in-gap Yu-Shiba-Rusinov states with an exotic spectrum whereas chains of adatoms result in a gapless Yu-Shiba-Rusinov band.

J. L. Lado et al. , 2D Materials 2016

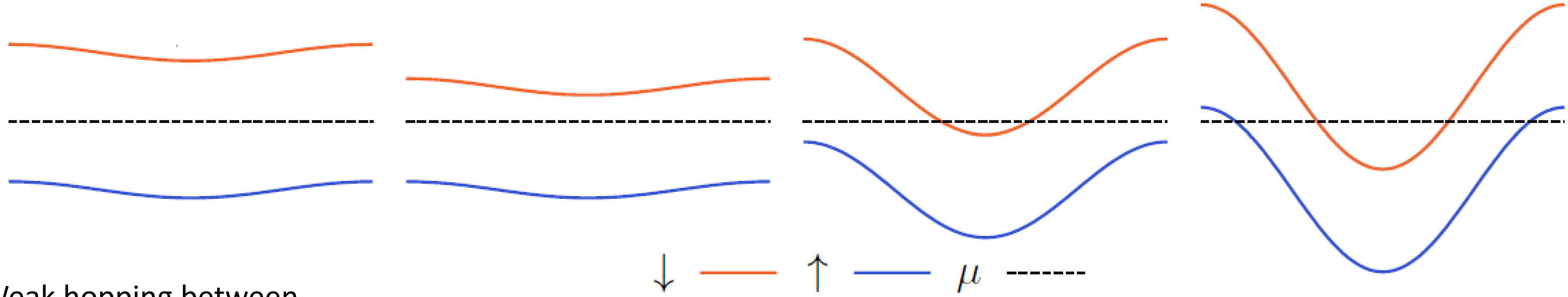
Yu-Shiba-Rusinov states



Array of magnetic impurities on an s-wave superconductor form a 2D Shiba lattice - system with high Chern numbers – large variety of topological orders.

J. Röntynen et al., PRL 2015

Chain of magnetic adatoms – different hopping strengths



Weak hopping between Anderson-impurity states which are symmetric about the Fermi energy. No electronic degrees of freedom

Weak hopping between generic Anderson-impurity states which are asymmetric about the Fermi energy.

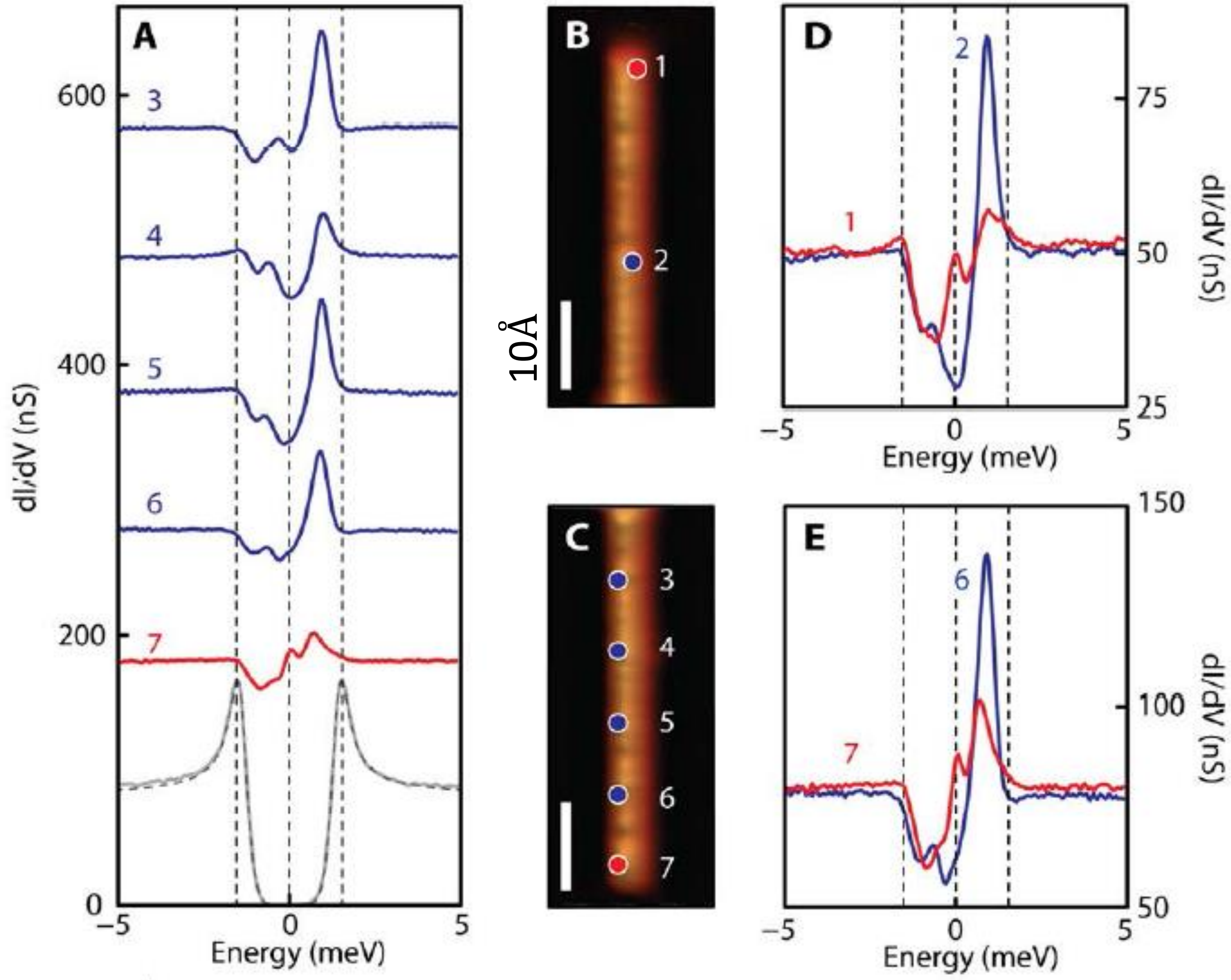
Strong hopping between Anderson impurity states.

Very strong hopping between Anderson impurity states.

The formation of Majorana bound states depends on the physics of the Shiba bands.

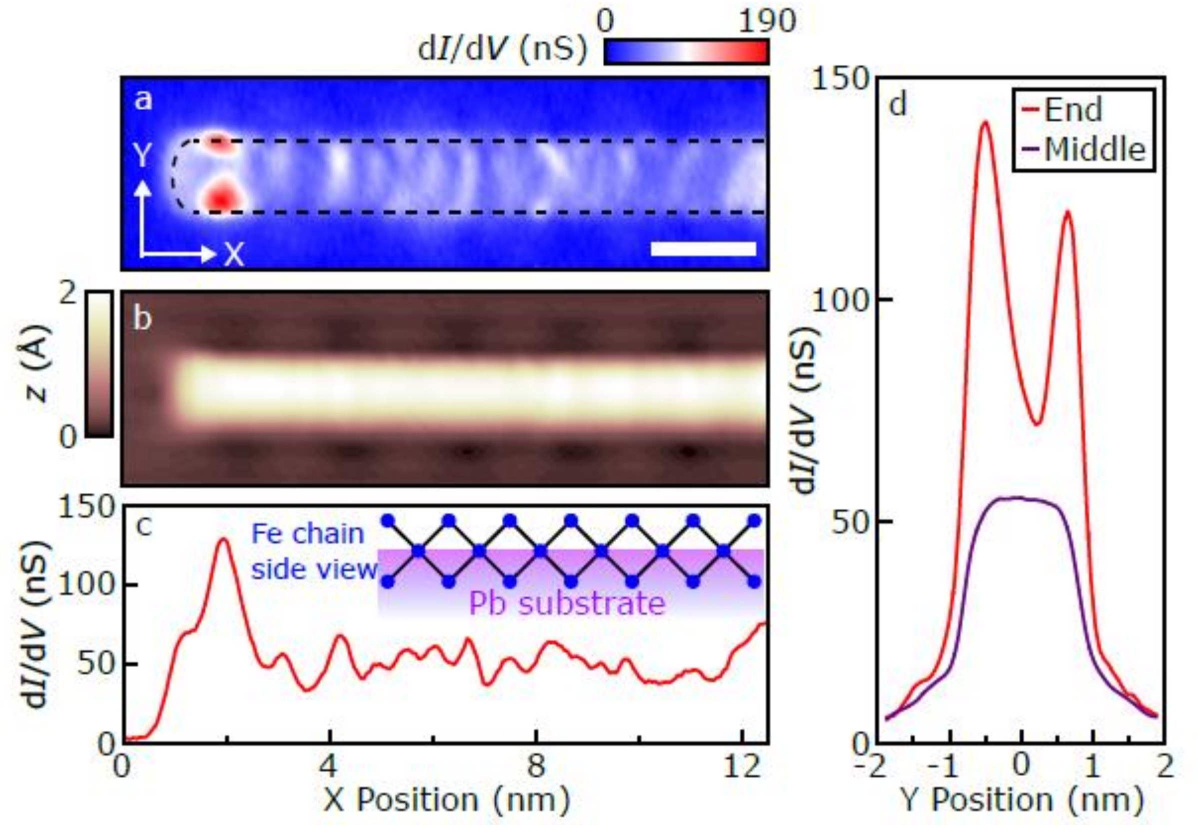
Prone to develop topological superconductivity in the presence of spin-orbit coupling in the superconductor.

Fe chain on Pb(110)



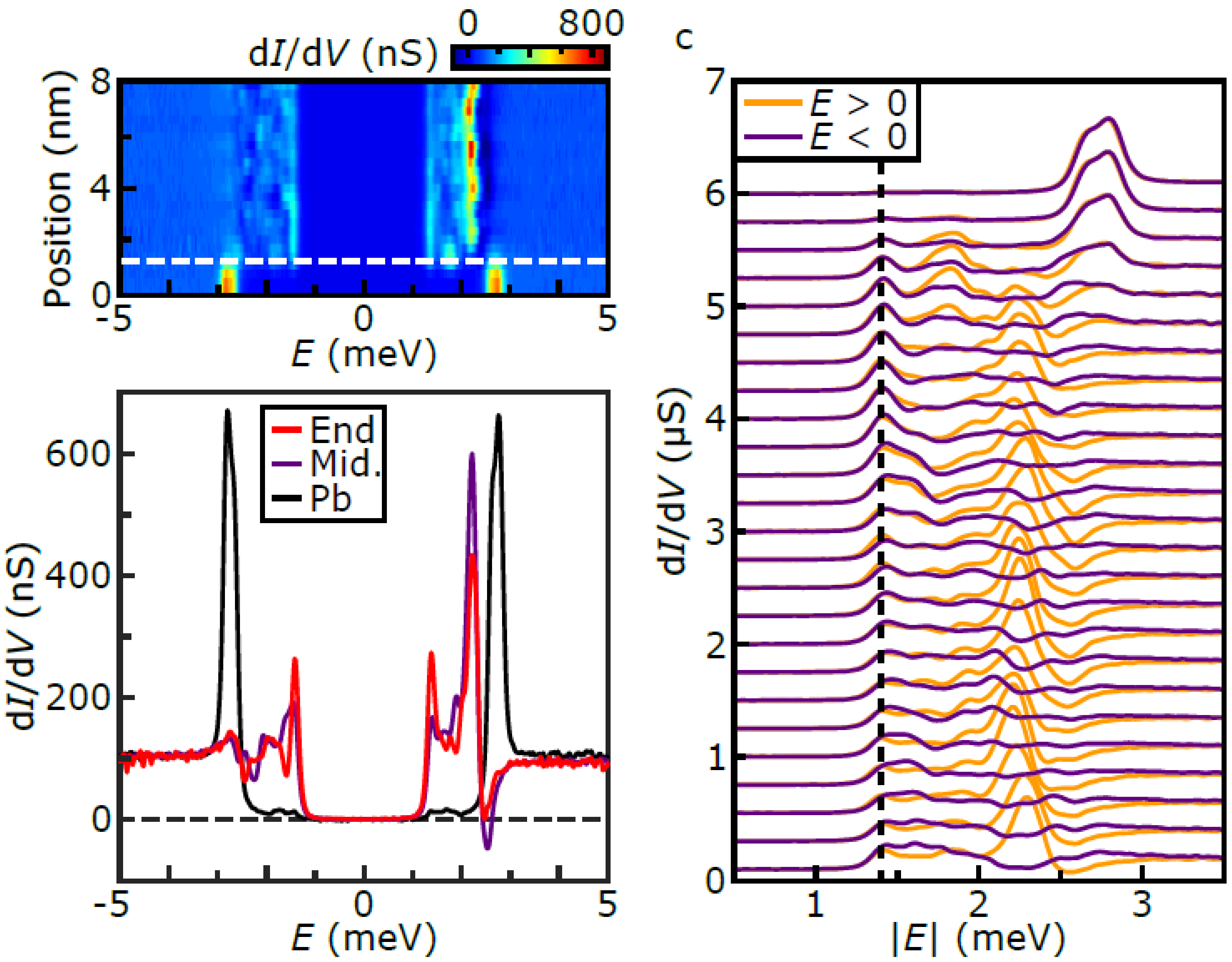
S. Nadj-Perge et al., Science 2014

Fe chain on Pb(110)



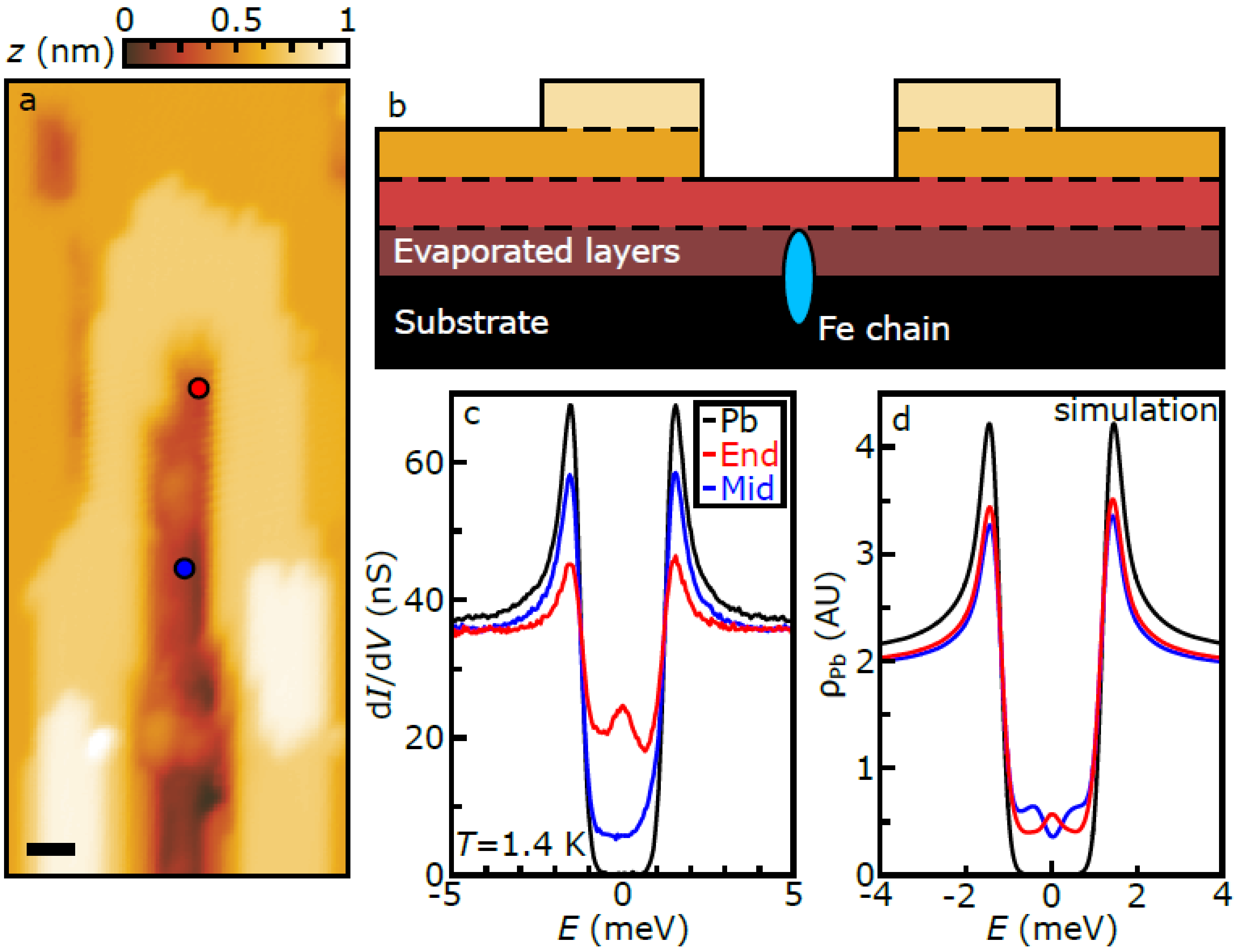
B. E. Feldman et al., Nature Physics 2017

Fe chain on Pb(110) – SC TIP



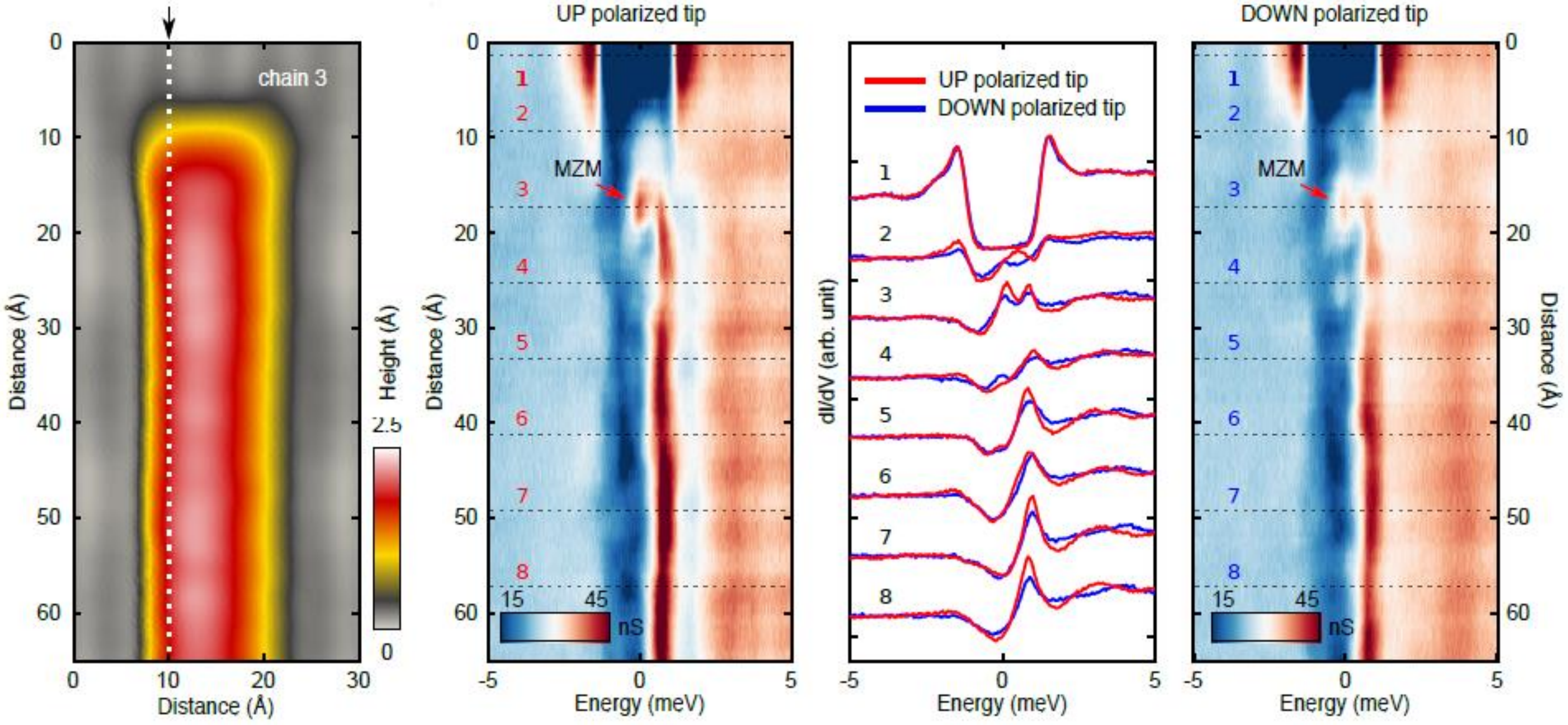
B. E. Feldman et al., Nature Physics 2017

Fe chain on & under Pb



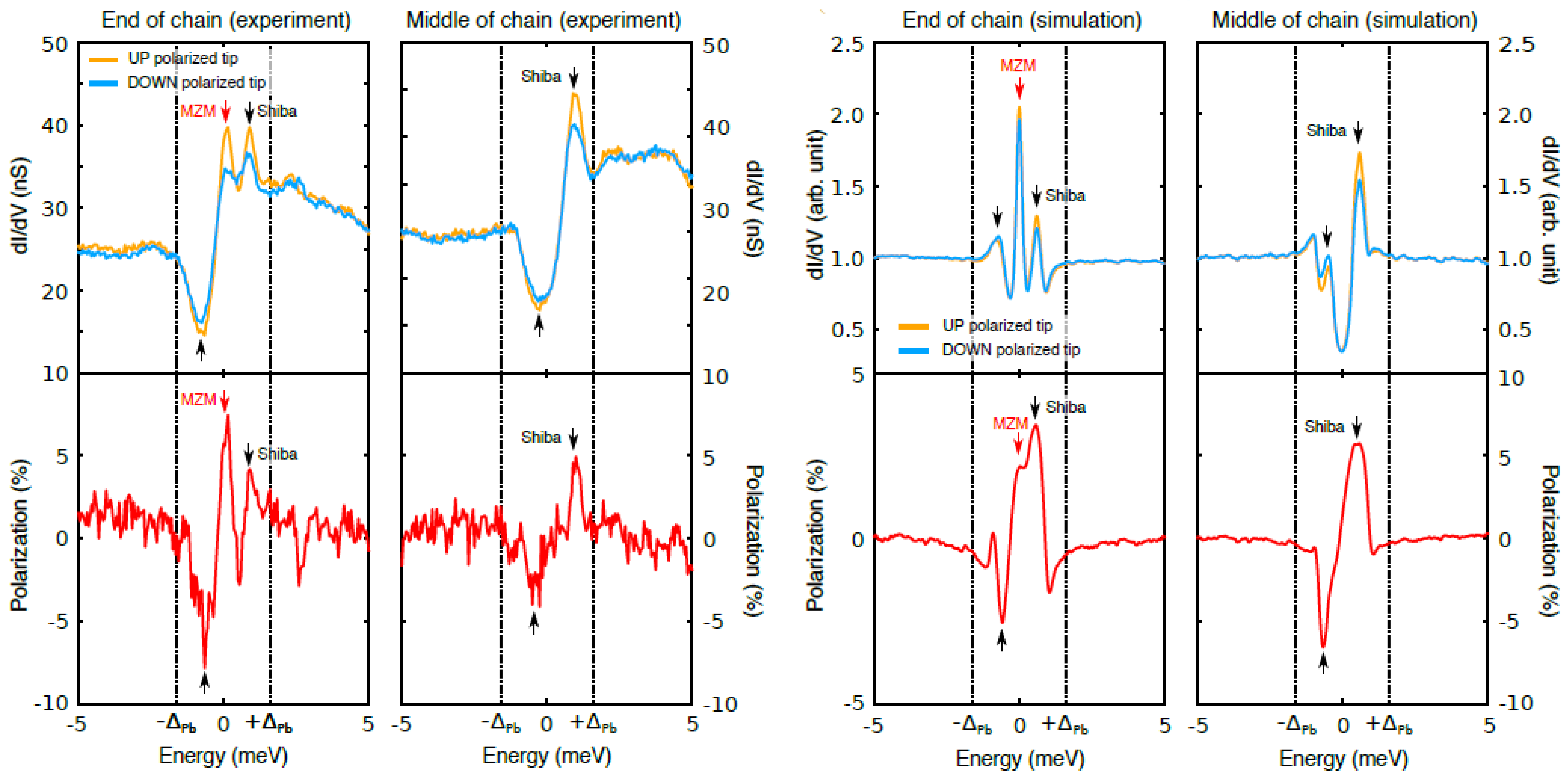
B. E. Feldman et al., Nature Physics 2017

Fe chain on Pb(110)



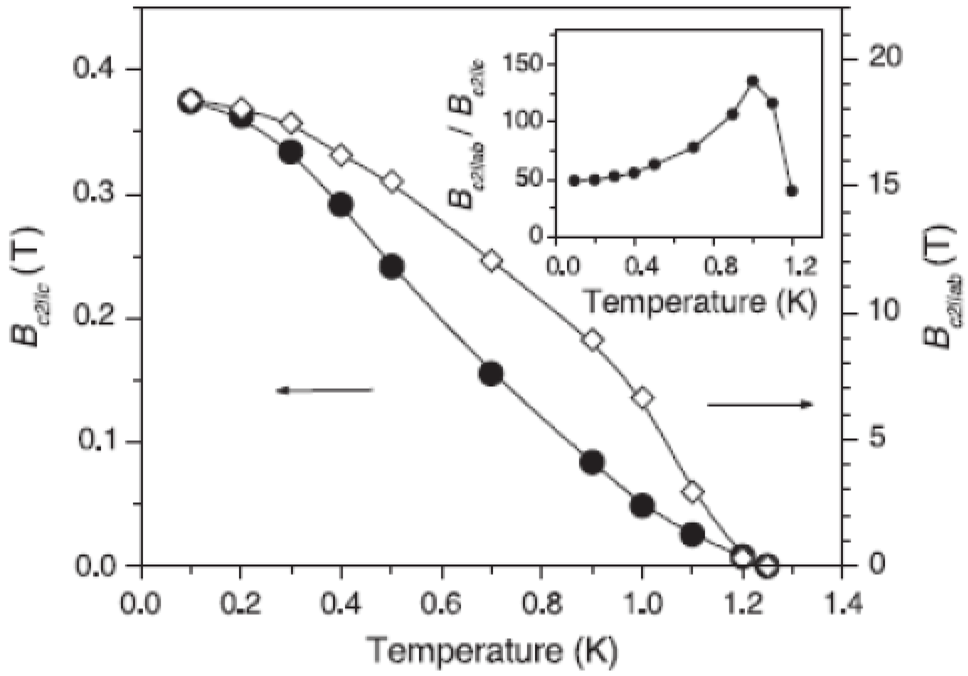
S. Jeon et al., Science 2017

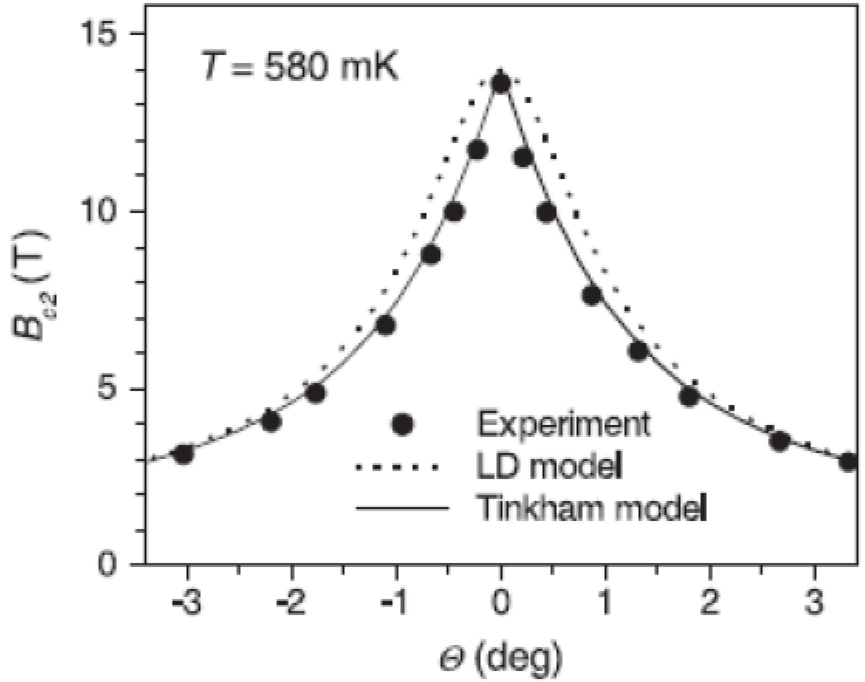
Fe chain on Pb(110)



S. Jeon et al., Science 2017

$(LaSe)_{1.14}(NbSe_2)$…Ising?





Upper critical magnetic field parallel to the basal planes (*ab*) exceeding by an order the Pauli limiting field together with B_{c2} in the *c*-direction. The inset shows the superconducting anisotropy indicating the 3D-2D crossover at 1.1 K

. Right: Angular dependence of B_{c2} in agreement with the Tinkham model for 2D superconductor

Bad news:

- Transformations realized by braiding of Majorana modes in 2D, while enjoying topological protection, cannot provide all three elementary gates required for a universal gate set.
- It is impossible to construct a universal topological quantum computer based on braiding Majorana bound states.

Two possible workarounds:

- complement braiding by unprotected gate operations – tedious but feasible
- Fibonacci anyons with a richer braid group and the capacity to realize a universal topological quantum computer - SCIFI