

Physics of heterostructure interfaces as a source for quantum information processing

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1st eduQUTE school on quantum technologies

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eduQUTE



Contents

- **Metallic junctions**

 - ***semiconductor / ferromagnetic interface***

 - spin-orbit coupling fields in solids

 - magnetic control of the fields

 - semiconducting wires & Majorana bound state story

- **van der Waals heterostructures**

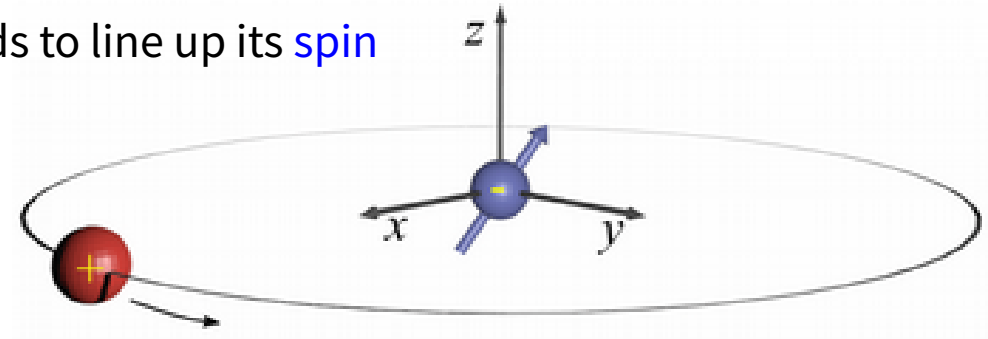
 - ***1D semiconductor / 2D superconductor interface***

 - proximity induced spin-orbit coupling effects

 - proposal Majorana bound states in carbon nanotubes

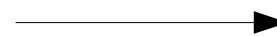
Spin-orbit coupling essentials

electron feels a magnetic field because of its own orbital motion
that tends to line up its **spin**

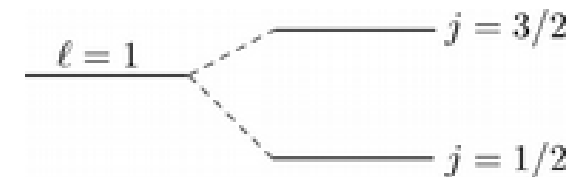


$$\mathcal{H}_{\text{so}} = \frac{\hbar}{4m^2c^2} (\nabla V(\mathbf{r}) \times \mathbf{p}) \cdot \boldsymbol{\sigma}$$

spin-orbit coupling field



$$\mathcal{H}_{\text{so}}^{\text{atom}} = \xi_{\ell} \mathbf{L} \cdot \mathbf{S}$$



communication between charge and spin => *spintronics*

Concept of spin-orbit coupling field in solids

time reversal + space inversion symmetry

$$\epsilon_{\mathbf{k},\uparrow} = \epsilon_{\mathbf{k},\downarrow}$$

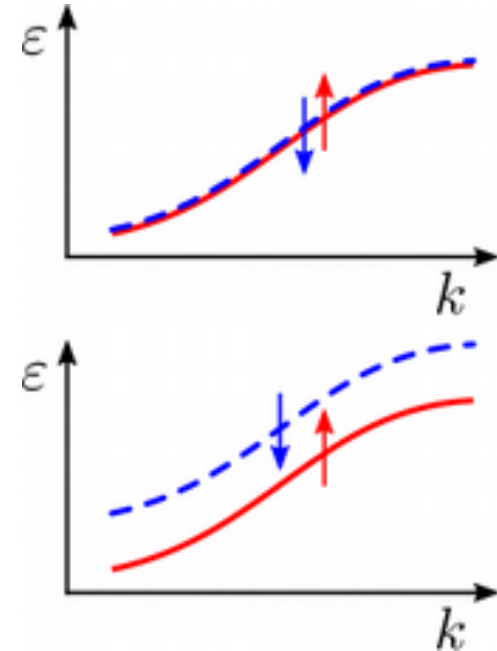
time reversal symmetry only

$$\epsilon_{\mathbf{k},\uparrow} = \epsilon_{-\mathbf{k},\downarrow}, \quad \epsilon_{\mathbf{k},\uparrow} \neq \epsilon_{\mathbf{k},\downarrow}$$

$$\Omega(-\mathbf{k}) = -\Omega(\mathbf{k})$$

*effective k-dependent
spin-splitting magnetic field*

$$\mathcal{H}_{\text{so}}(\mathbf{k}) = \frac{\hbar}{2} \Omega(\mathbf{k}) \cdot \sigma$$



$$\mathcal{H}_{\text{D}}(\mathbf{k}) = \gamma(\sigma_y k_y - \sigma_x k_x) \longrightarrow \Omega_{\text{BIA}}(\mathbf{k}) = \gamma(-k_x, k_y)$$

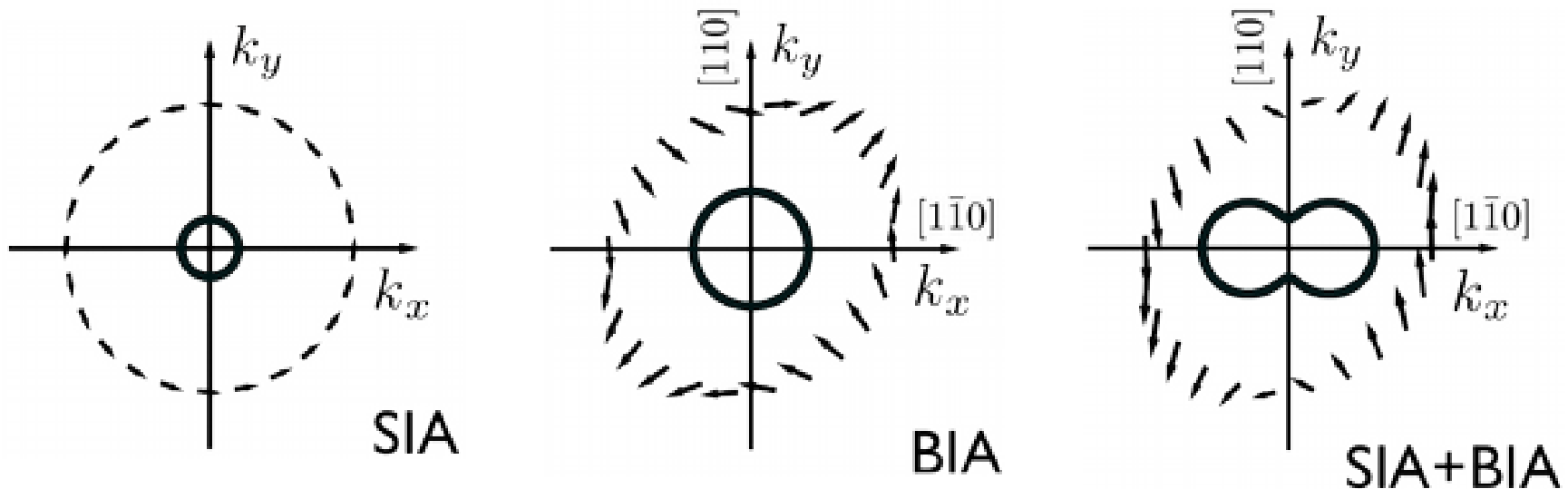
bulk inversion asymmetry (BIA)

$$\mathcal{H}_{\text{BR}}(\mathbf{k}) = \alpha(\sigma_y k_x - \sigma_x k_y) \longrightarrow \Omega_{\text{SIA}}(\mathbf{k}) = \alpha(-k_y, k_x)$$

structure inversion asymmetry (SIA)

Combination of the linearized spin-orbit fields

symmetry lowering



$$\mathcal{H}_D(\mathbf{k}) = \gamma(\sigma_y k_y - \sigma_x k_x) \quad \longrightarrow \quad \Omega_{\text{BIA}}(\mathbf{k}) = \gamma(-k_x, k_y)$$

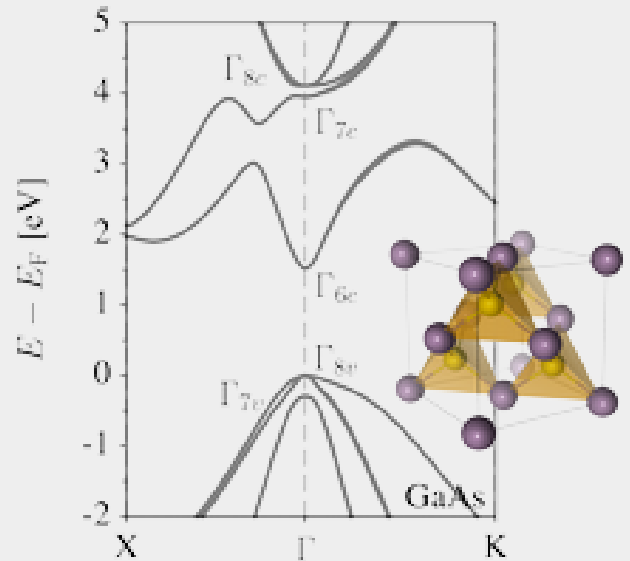
bulk inversion asymmetry (BIA)

$$\mathcal{H}_{\text{BR}}(\mathbf{k}) = \alpha(\sigma_y k_x - \sigma_x k_y) \quad \longrightarrow \quad \Omega_{\text{SIA}}(\mathbf{k}) = \alpha(-k_y, k_x)$$

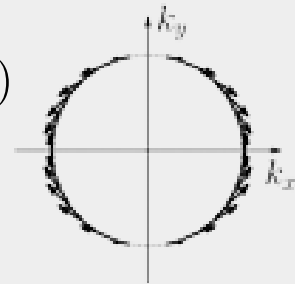
structure inversion asymmetry (SIA)

SOC fields in bulk III-V semiconductors

zinc-blende



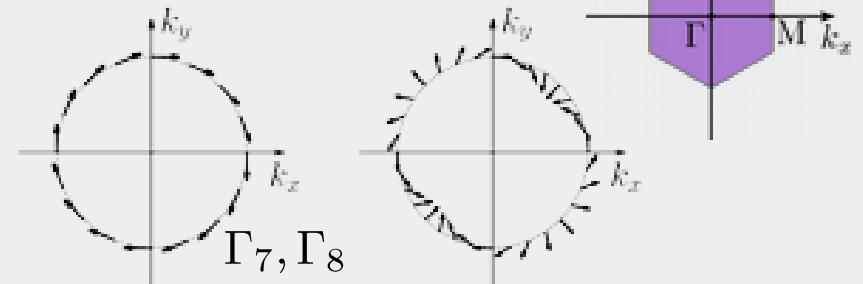
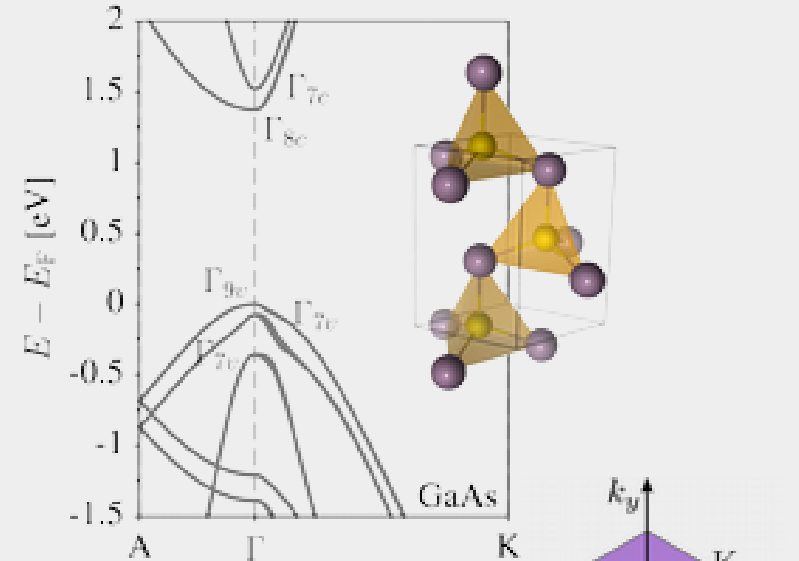
$$\Omega(k_x, k_y, 0)$$



$$\Omega(\mathbf{k})^{\Gamma_6} = \gamma[k_x(k_y^2 - k_z^2), k_y(k_z^2 - k_x^2), k_z(k_x^2 - k_y^2)]$$

G. Dresselhaus, Phys. Rev. **100**, 580 (1955)

wurtzite



$$\Omega(\mathbf{k})^{\Gamma_{7/8}} = (\alpha + \gamma[bk_z^2 - k_{\parallel}^2])(k_y, -k_x, 0)$$

$$\Omega(\mathbf{k})^{\Gamma_9} = \gamma(k_y(k_y^2 - 3k_x^2), k_x(k_x^2 - 3k_y^2), 0)$$

J. Y. Fu and M. W. Wu, JAP **104**, 093712 (2008)

Spin-orbit coupling parameters for bulk III-V

extracted parameters for conduction band

	GaAs	GaSb	InAs	InSb
zinc-blende				
Γ_{6c}				
$\mathbf{\Omega}(\mathbf{k}) = \gamma[k_x(k_y^2 - k_z^2), k_y(k_z^2 - k_x^2), k_z(k_x^2 - k_y^2)]$				
γ [eV Å ³]	9.13	105.3	21.4	200
	(8.5)	(119)	(47)	(209)
	[a]		[b]	[b]
wurtzite				
Γ_{7c}				
$\mathbf{\Omega}(\mathbf{k}) = (\alpha + \gamma[bk_z^2 - k_{ }^2])(k_y, -k_x, 0)$				
α [eV Å]	0.04	0.078	0.3	0.76
γ [eV Å ³]	6.51	52.1	134.2	904
b	0.54	1.29	-1.25	-0.93
Γ_{8c}				
$\mathbf{\Omega}(\mathbf{k}) = (\alpha + \gamma[bk_z^2 - k_{ }^2])(k_y, -k_x, 0)$				
α [eV Å]	0.1	0.49	0.04	0.34
γ [eV Å ³]	1.97	18.7	2.77	10.9
b	0.03	-0.04	-0.06	-0.07

[a] M.I. McMahon, R.J. Nemes, Phys. Rev. Lett. **95**, 215505 (2005)

[b] D. Kriegner *et al.*, Nano Lett. **11**, 1438 (2011)

[*] A.N. Chantis *et al.*, Phys. Rev. Lett. **96**, 086406 (2006)

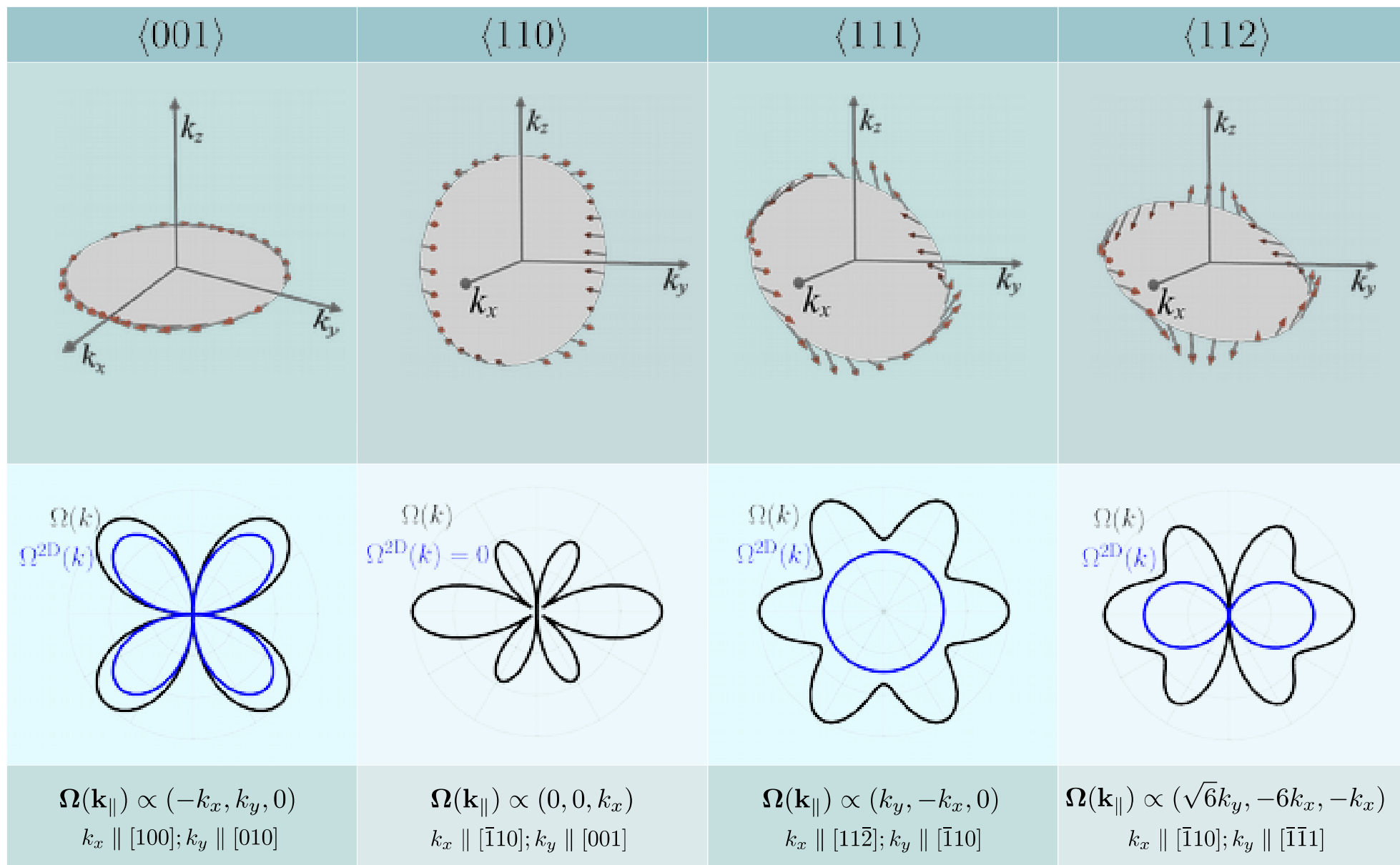
M. Gmitra, J. Fabian, Phys. Rev. B **94**, 165202 (2016)

DFT calculations
WIEN2k + mBJ



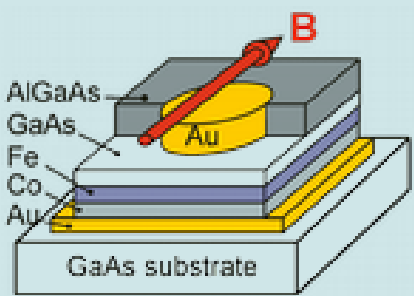
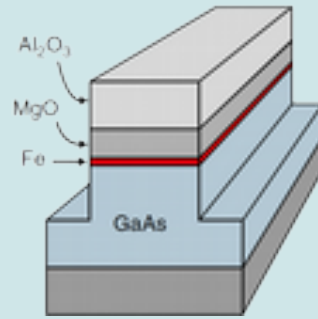
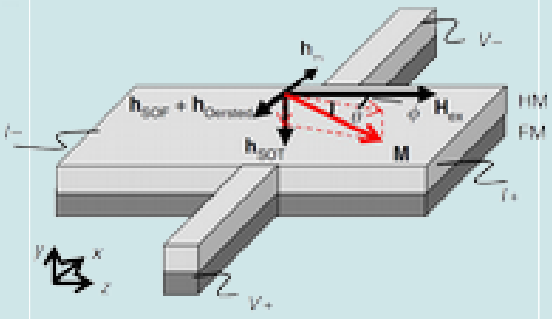
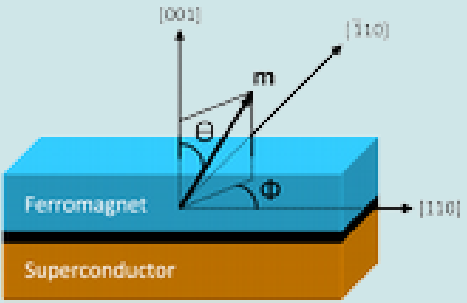
Spin-orbit fields in zinc-blende

growth directions of quantum wells



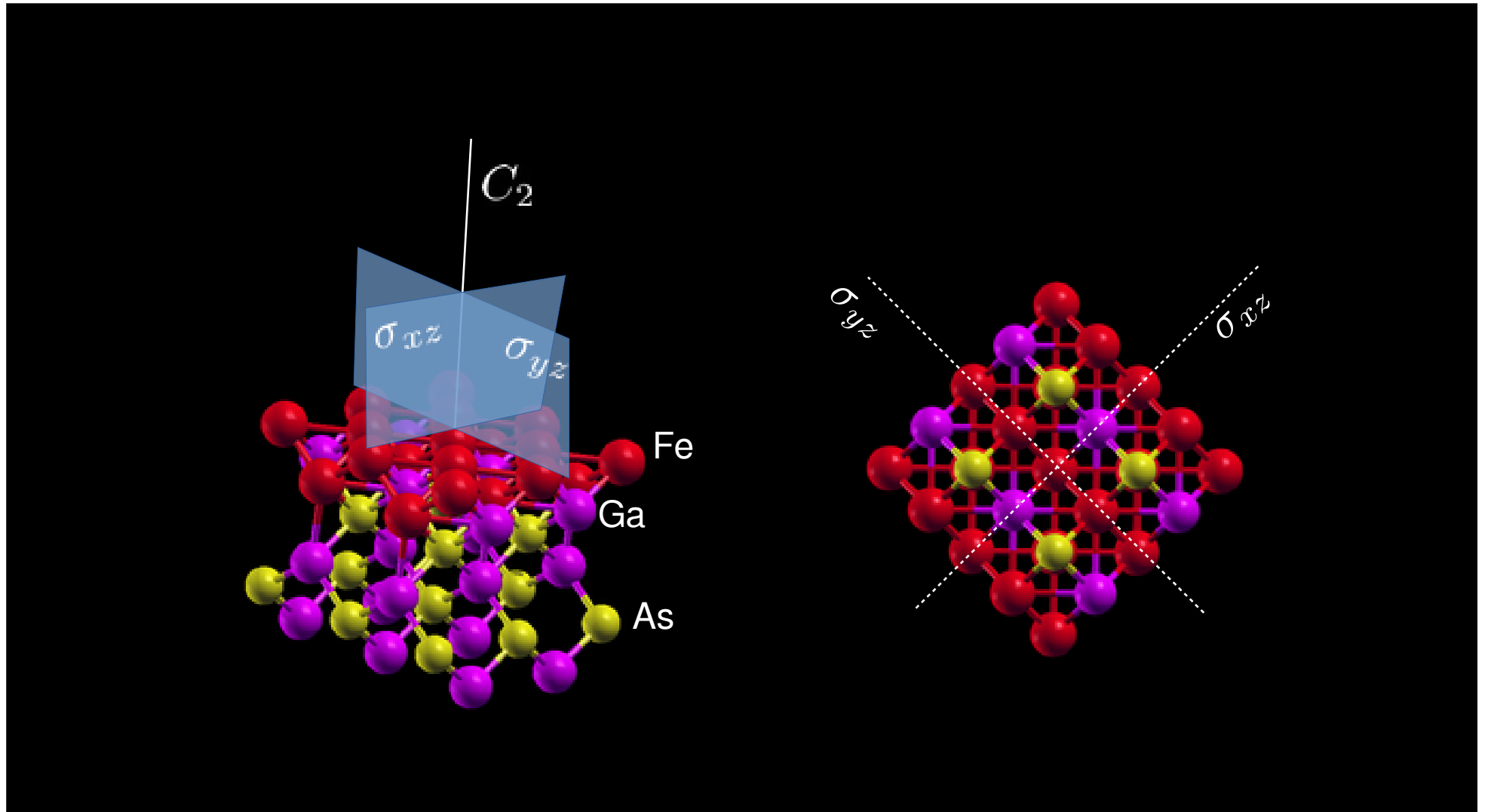
Manifestation of spin-orbit fields

magnetoresistive effects

<p>GMR/TMR (<i>giant magnetoresistance</i>)</p>	<p>GMR/TMR (<i>giant magnetoresistance</i>)</p>	<p>Hall effect</p>	<p>SPAR (<i>spin polarized Andreev reflection</i>)</p>
<p>TAMR (<i>tunneling anisotropic magnetoresistance</i>)</p>	<p>CAMR (<i>crystalline anisotropic magnetoresistance</i>)</p>	<p>PHE (<i>planar Hall effect</i>)</p>	<p>MAAR (<i>magnetoanisotropic Andreev reflection</i>)</p>
			
<p>J. Moser <i>et al.</i>, PRL 99, 056601 (2007)</p>	<p>T. Hupfauer <i>et al.</i>, Nat. Comm. 6, 7374 (2015)</p>	<p>X. Fan <i>et al.</i>, Nat. Comm. 4, 1799 (2013)</p>	<p>P. Högl <i>et al.</i>, PRL 115, 116601 (2015)</p>

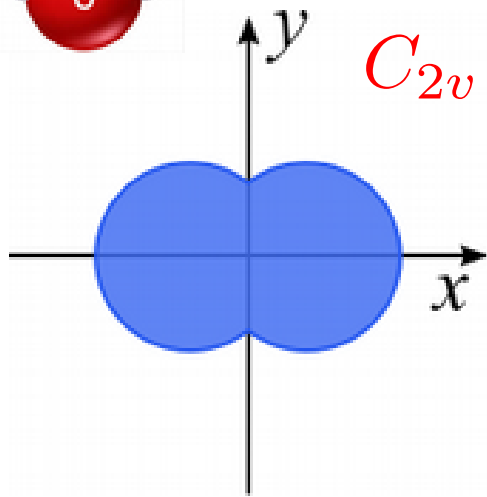
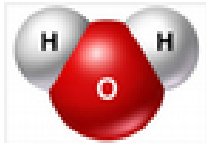
Symmetry of Fe/GaAs (001) interface

advances of epitaxial growth



Exploring symmetry

a path to the effective Hamiltonian



$$C_{2v} : E, C_2, \sigma_{xz}, \sigma_{yz}$$

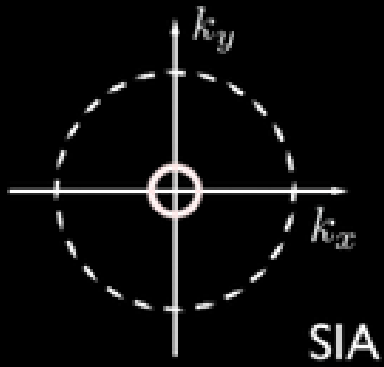
$$H_{\text{plane}} \sim (\alpha x^2 + \beta y^2) \times z$$

$$k_x \sim x, \quad k_y \sim y, \quad \sigma_x \sim yz, \quad \sigma_y \sim -xz$$

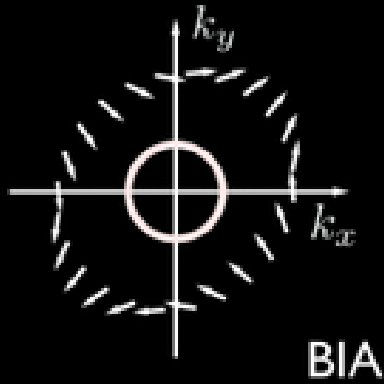
$$H_{so} \sim \alpha k_x \sigma_y + \beta k_y \sigma_x$$

generic linear in k spin-orbit coupling of C_{2v}

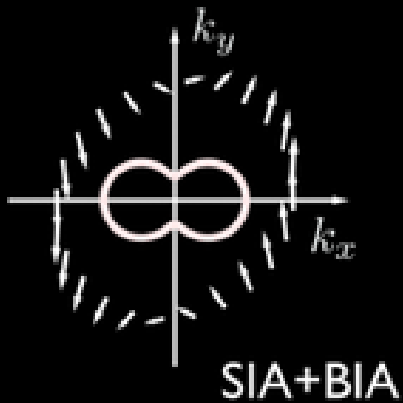
Bychkov-Rashba



+ Dresselhaus

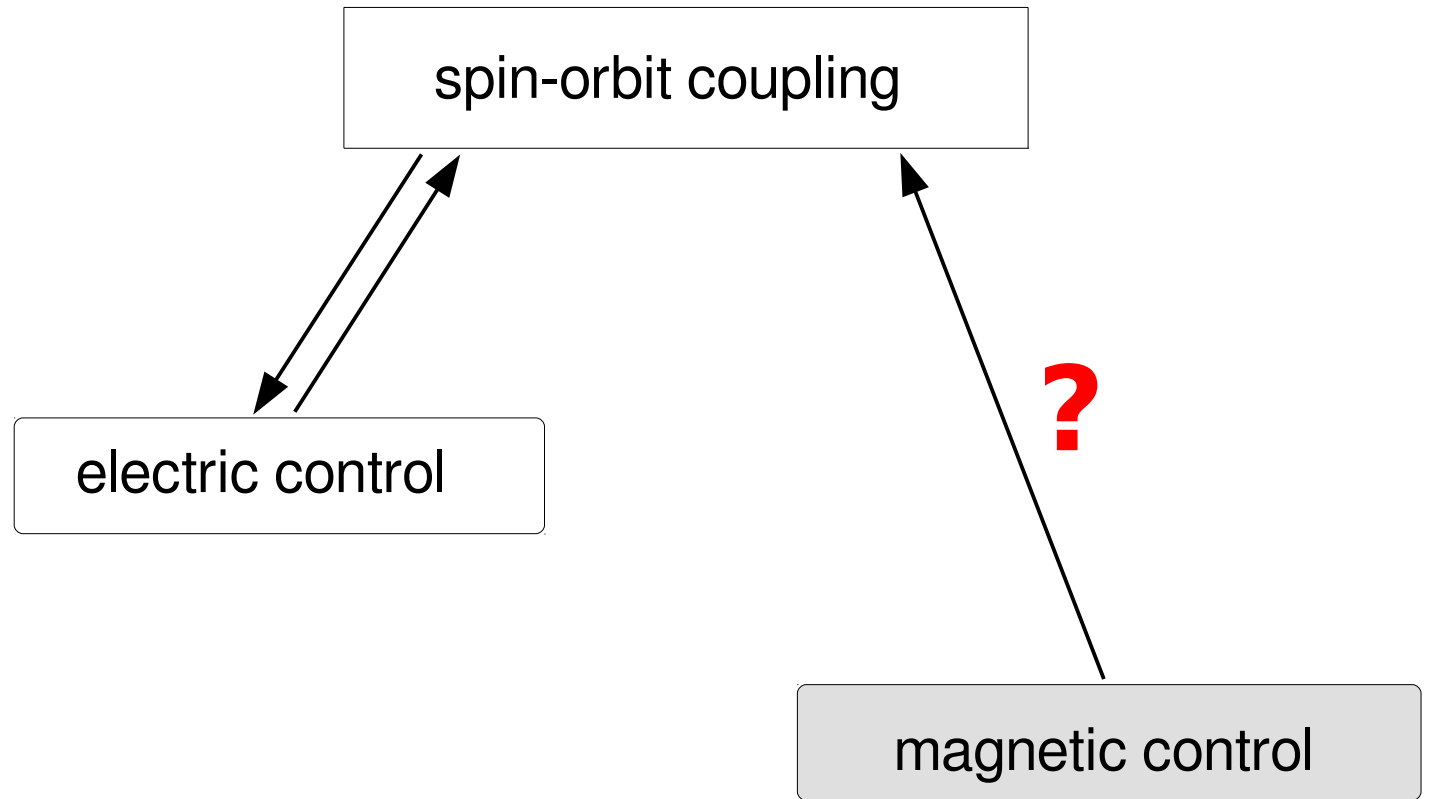


= C_{2v} symmetry



Fe/GaAs (001) interface

change of the paradigm



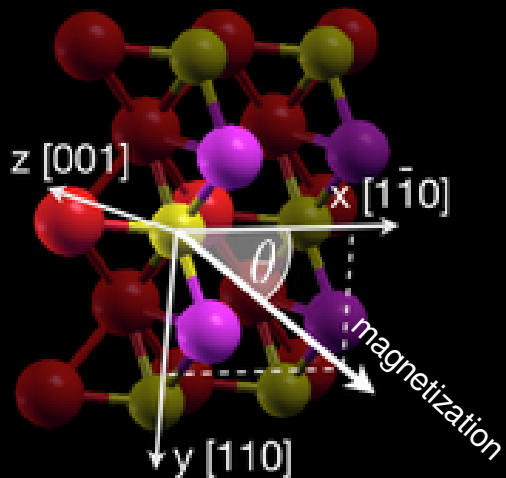
Parameters depend on magnetization

most general Hamiltonian for C_{2v}

$$\mathcal{H}_{\text{SO}} = \mu_n(k_x, k_y, \theta) k_x \sigma_y + \eta_n(k_x, k_y, \theta) k_y \sigma_x$$

$$\mu_n(k_x, k_y, \theta) = \mu_n^{(0)}(\theta) + \mu_n^{(1)}(\theta) k_x^2 + \mu_n^{(2)}(\theta) k_y^2 + \dots$$

$$\eta_n(k_x, k_y, \theta) = \eta_n^{(0)}(\theta) + \eta_n^{(1)}(\theta) k_x^2 + \eta_n^{(2)}(\theta) k_y^2 + \dots$$

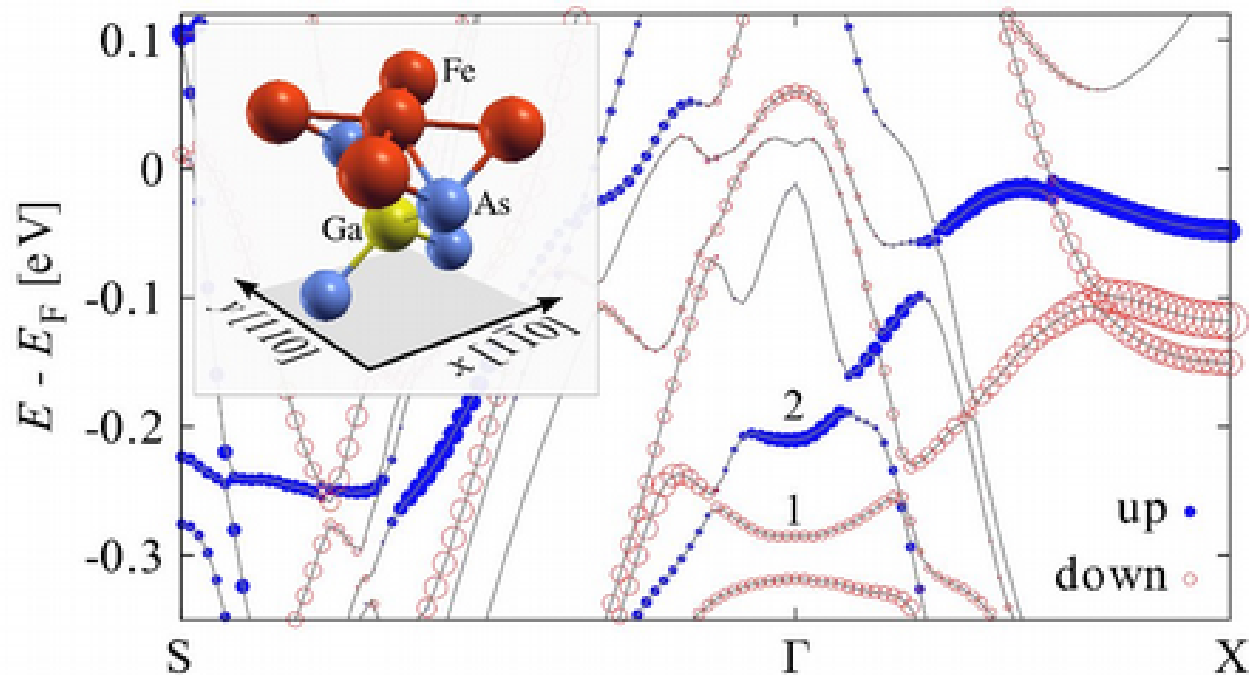


$$\Omega(k_x, k_y, \theta) = \begin{pmatrix} \eta_n(k_x, k_y, \theta) \\ \mu_n(k_x, k_y, \theta) \\ 0 \end{pmatrix}$$

Spin-orbit field obtained from bands

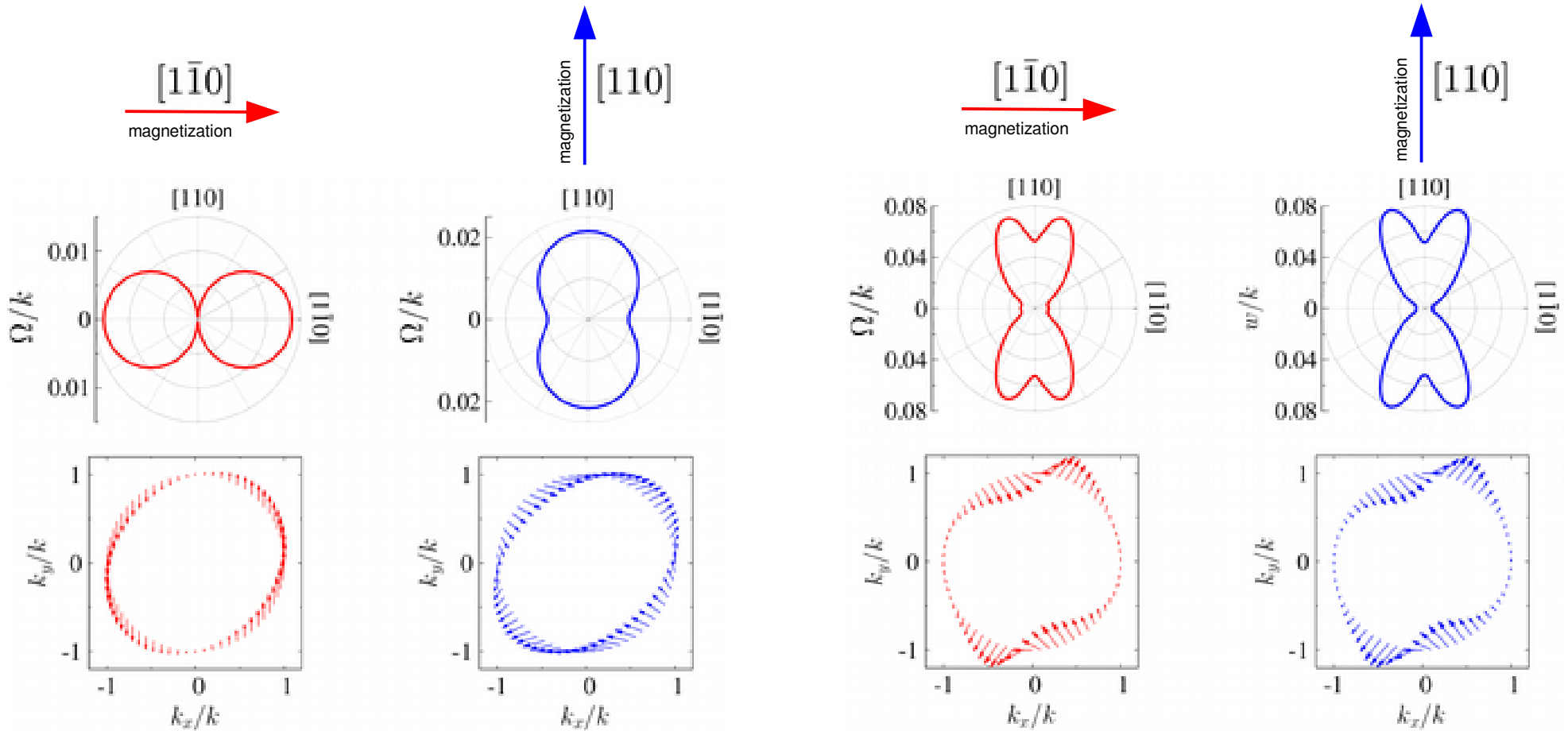
$$\Omega_{nx}(k_x, k_y, \theta) = \sigma \left[\frac{E_n(\mathbf{k}, \theta) - E_n(-\mathbf{k}, \theta) + E_n(-k_x, k_y, \theta) - E_n(k_x, -k_y, \theta)}{4 \cos \theta} \right]$$

$$\Omega_{ny}(k_x, k_y, \theta) = \sigma \left[\frac{E_n(\mathbf{k}, \theta) - E_n(-\mathbf{k}, \theta) - E_n(-k_x, k_y, \theta) + E_n(k_x, -k_y, \theta)}{4 \sin \theta} \right]$$



Magnetic control of spin-orbit symmetry

spin-orbit field for general k-point



$k = 1\%BZ$

$k = 12.5\%BZ$

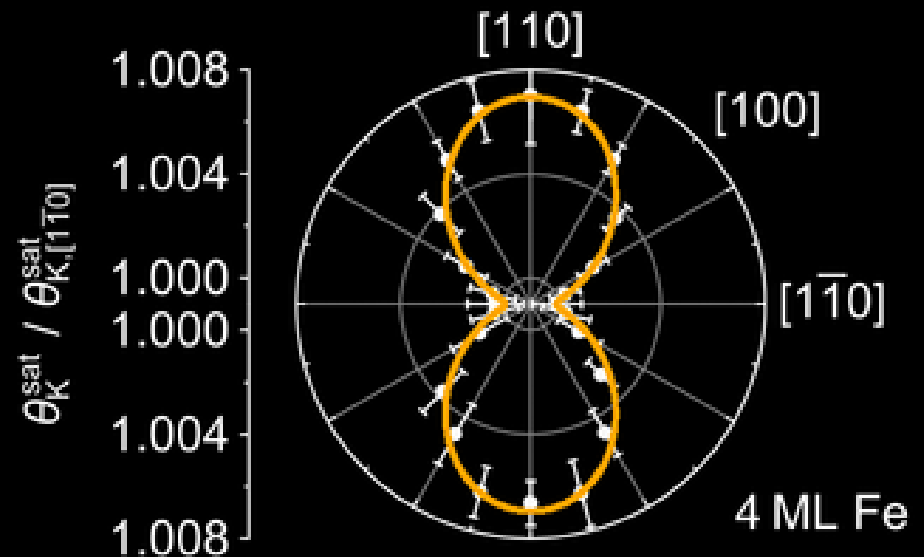
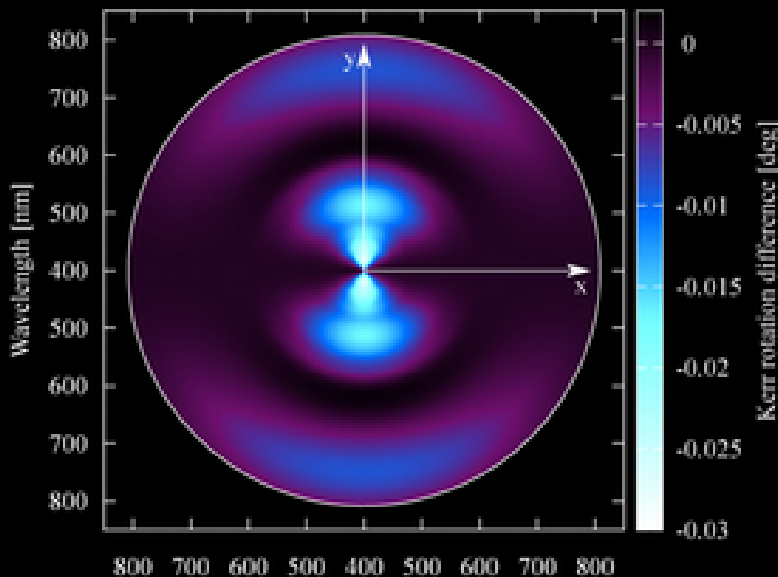
Symmetry of Fe/GaAs (001) interface

addressing the spin-orbit field by optics

anisotropic polar MOKE – Kerr rotation

$$C_{2v} : E, C_2, \sigma_{xz}, \sigma_{yz}$$

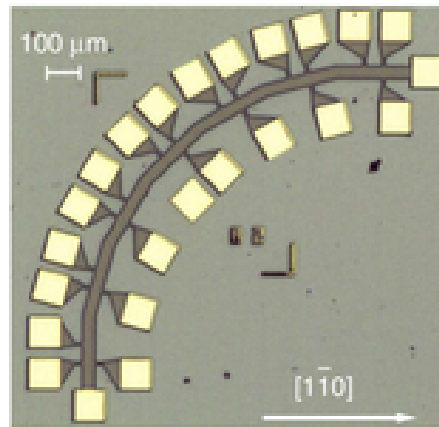
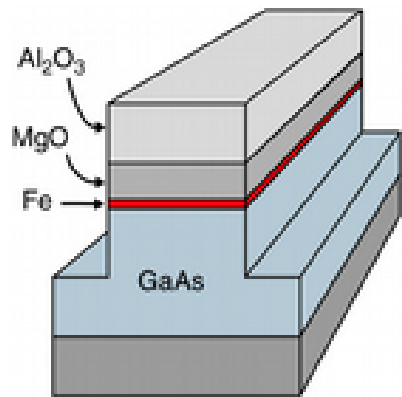
$$\frac{\theta_K^{\text{sat}}(\phi)}{\theta_{K,[1\bar{1}0]}^{\text{sat}}} - 1 \sim \langle a_2^{(2)}(\mathbf{k}) w_x(\mathbf{k}) w_y(\mathbf{k}) \rangle [1 - \cos(2\phi)]$$



S. Putz *et al.*, Phys. Rev. B **90**, 045315 (2014)

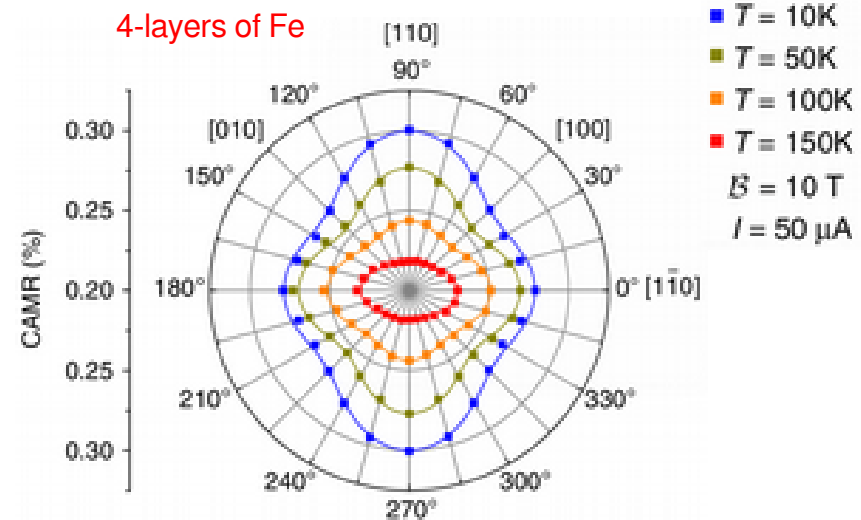
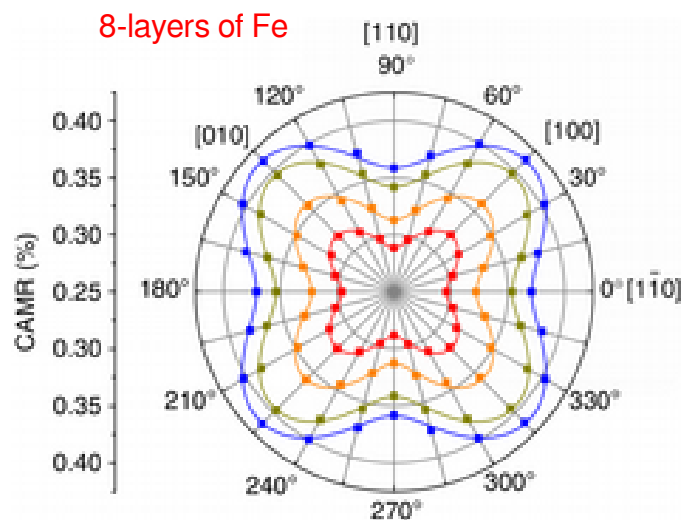
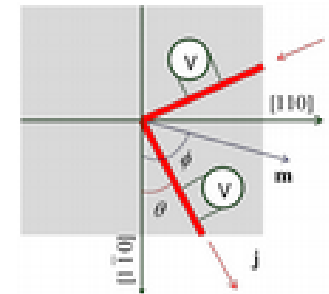
M. Buchner *et al.*, Phys. Rev. Lett. **117**, 157202 (2016)

Crystalline anisotropic magnetoresistance



$$\text{CAMR}(\theta) = \frac{U_{\max}(\theta) - U_{\min}(\theta)}{U_{\max}(\theta) + U_{\min}(\theta)}$$

$$\text{CAMR}(\theta) \approx \frac{B + C + (C - B)\cos(4\theta) - 4F\cos(2\theta)}{4A}$$



- $T = 10\text{K}$
- $T = 50\text{K}$
- $T = 100\text{K}$
- $T = 150\text{K}$
- $B = 10\text{ T}$
- $I = 50\ \mu\text{A}$

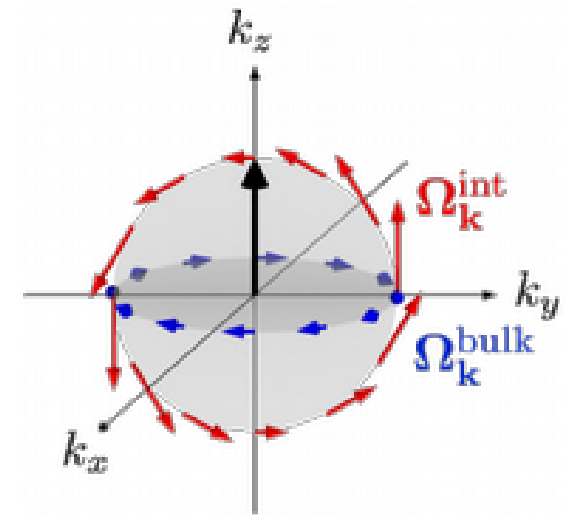
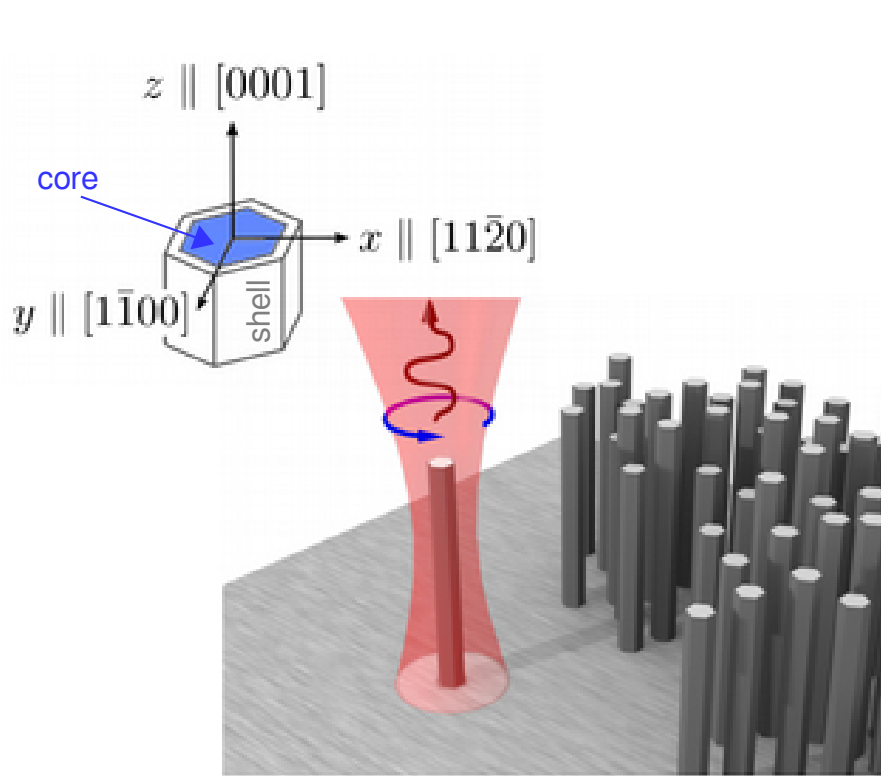
reduction of Fe thickness

reduction of symmetry

T. Hupfauer *et al.*, Nat. Comm. **6**, 7374 (2015)

Spin-orbit field in core/shell wurtzite nanowires

role of GaAs/AlGaAs interface

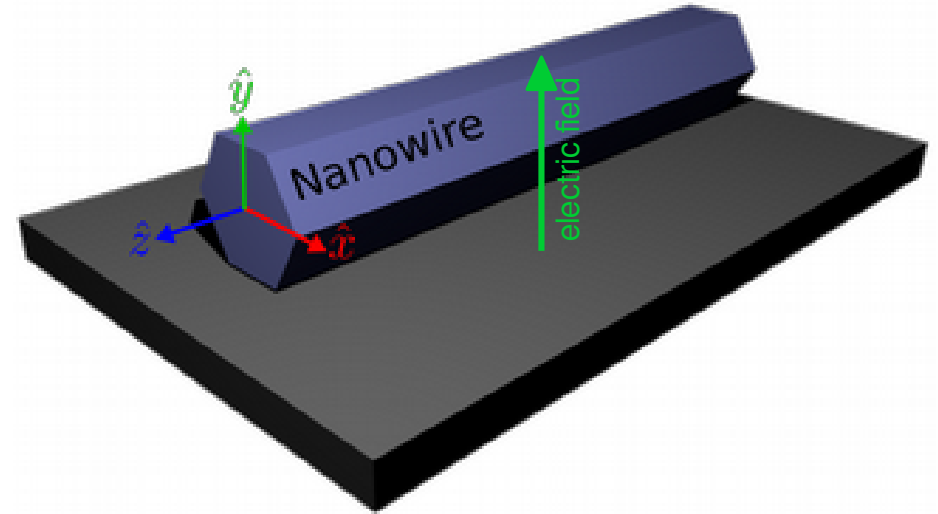
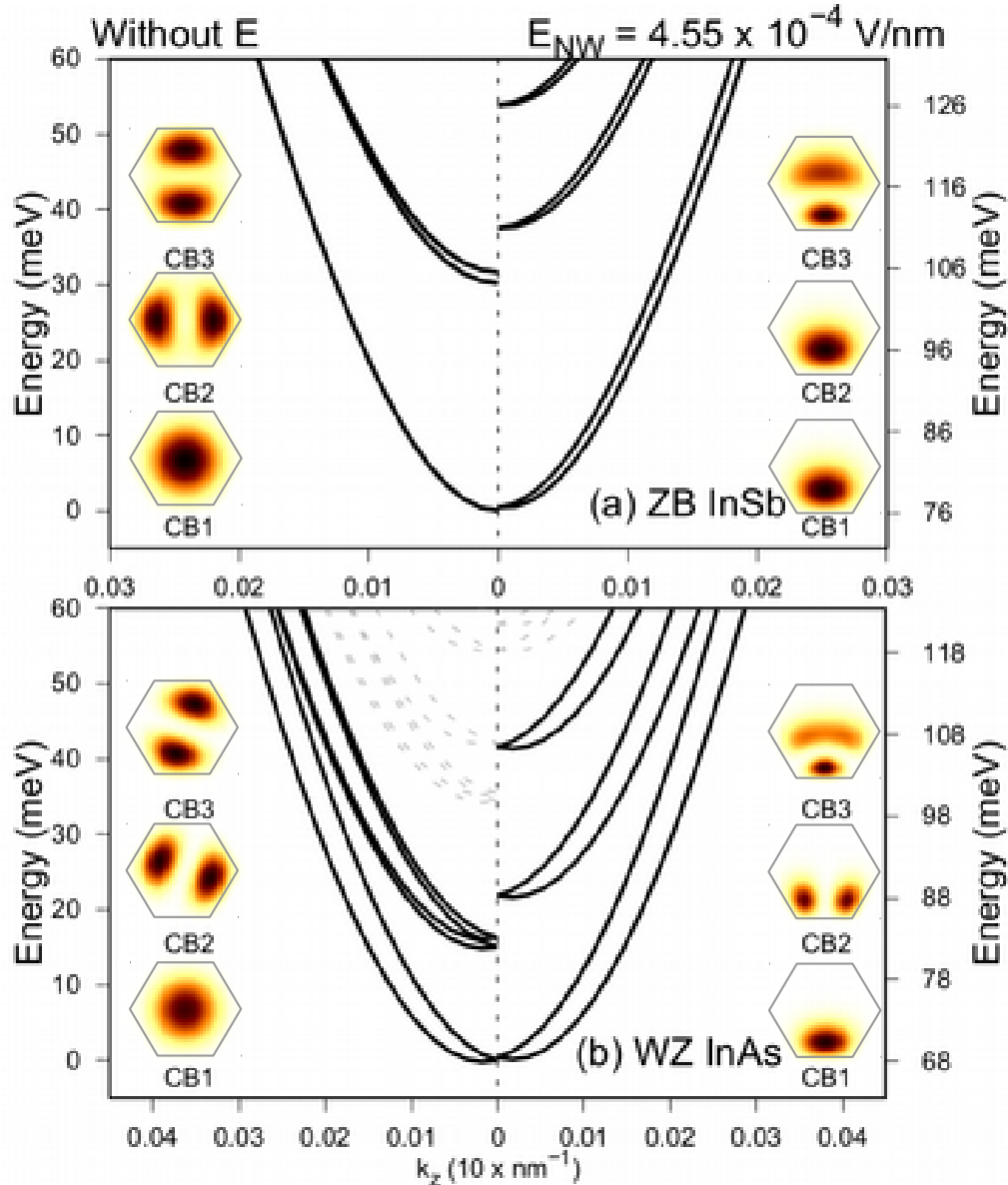


$$\Omega_{\mathbf{k}} = \Omega_{\mathbf{k}}^{\text{bulk}} + \Omega_{\mathbf{k}}^{\text{int}} = \beta \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\alpha_{\perp} k_z \\ \alpha_{\parallel} k_y \end{pmatrix}$$

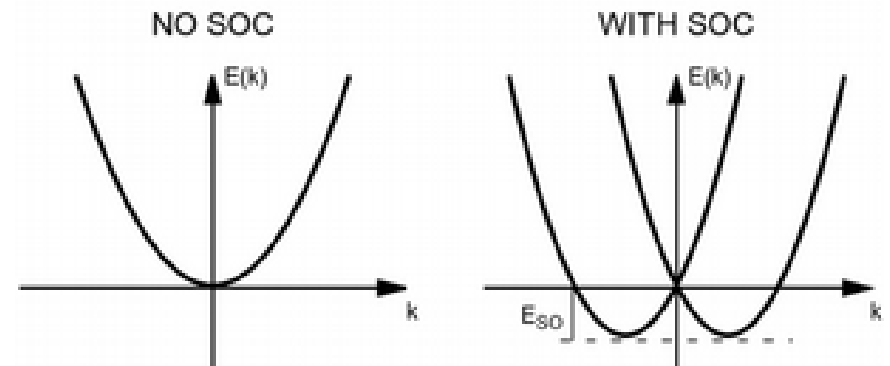
- pioneering optical spin injection into a **single** free-standing nanowire
- **g -factor** significantly **different** than in the cubic zinc-blende phase
- highly **anisotropic spin relaxation** due to SOC at core/shell interface

Conduction subbands in nanowires (50 nm)

an element for Topological Quantum Computation

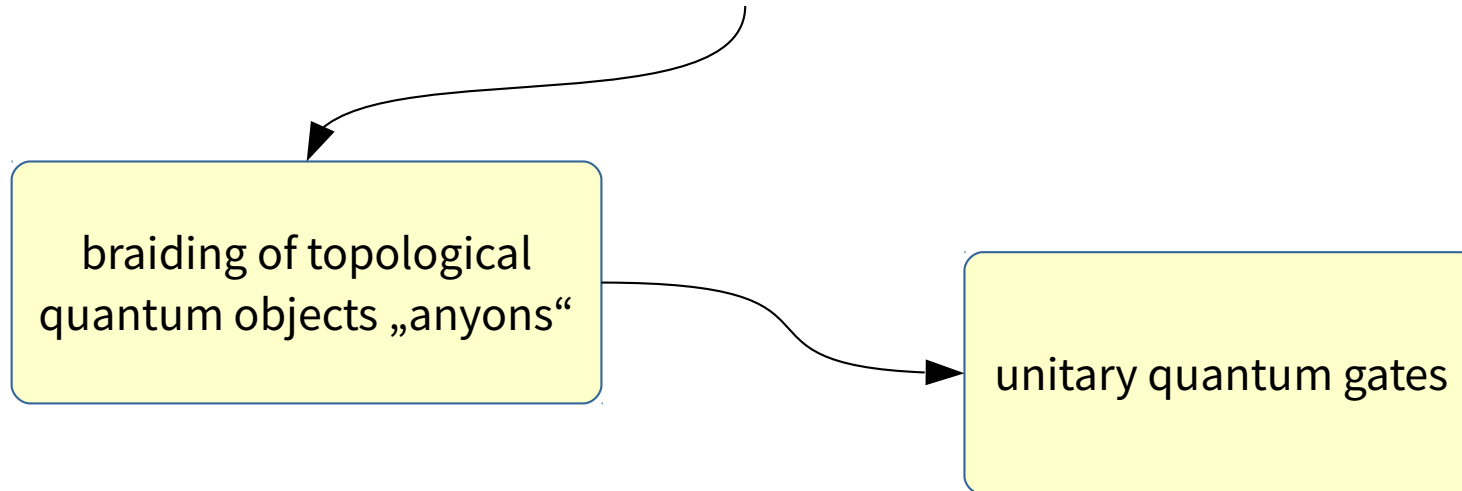


T. Campos *et al.*, arXiv: 1802.06734



Topological Quantum Computation

approach to fault-tolerant quantum computation



- **Localized excitations on an interacting Hamiltonian**
(Laughlin fractional Quantum Hall liquid)
- **Defects in an ordered system**
(Abrikosov vortices in topological superconductor / domain wall in 1D system)

simplest realization of non-Abelian anyons (no anionic excitations) is a quasiparticle or defect supporting a **Majorana zero mode** (zero energy mid-gap excitation)

What is the Majorana zero mode?

zeroth order crash course

- **is a fermionic operator**

- **squares to unity**

$$\left. \begin{array}{l} \gamma \\ \gamma^2 = 1 \end{array} \right\} \text{Majorana fermion}$$

- **commutes with the Hamiltonian of a system**

$$[\gamma, H] = 0$$

- **degenerate ground state / non-local entanglement**

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

- **decomposition to conventional fermions**

$$c_j = \frac{1}{2}(\gamma_{2j-1} + i\gamma_{2j})$$

To realize Majorana excitations $\gamma = \gamma^\dagger$ we seek for, e. g., $\gamma = uc + vc^\dagger$ with energy dependent coefficients of Bogoliubov quasiparticle excitations

BCS **spinless** fermion pairing \rightarrow Cooper pair wave function must be **antisymmetric** therefore Majorana zero-energy excitations should exist in **p-wave superconductors**

Spinless p -wave superconductors

continuum mean-field many-particle Hamiltonian

$$\mathcal{H} = \int dx \left\{ \psi^\dagger(x) \left(\frac{p^2}{2m} - \mu \right) \psi(x) + \Delta' [\psi^\dagger(x) \partial_x \psi^\dagger(x) + \text{h.c.}] \right\}$$

associated Bogoliubov–de Gennes Hamiltonian

$$H = \begin{pmatrix} \xi_p & -i\Delta' p \\ i\Delta' p & -\xi_p \end{pmatrix} = \xi_p \tau_z + \Delta' p \tau_y \quad \text{where} \quad \xi_p = p^2/2m - \mu$$

excitation spectrum for an infinite system

$$E_k = \pm (\xi_k^2 + \Delta'^2 k^2)^{1/2}$$

effective BdG field in particle-hole space

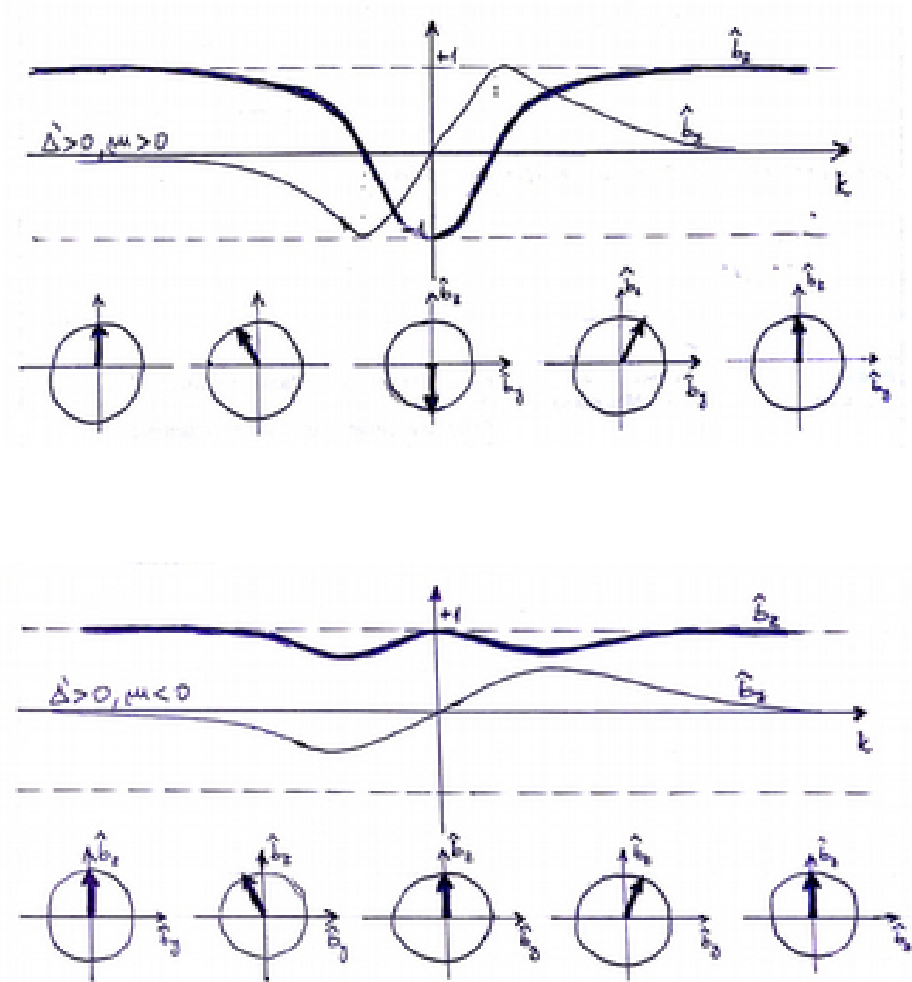
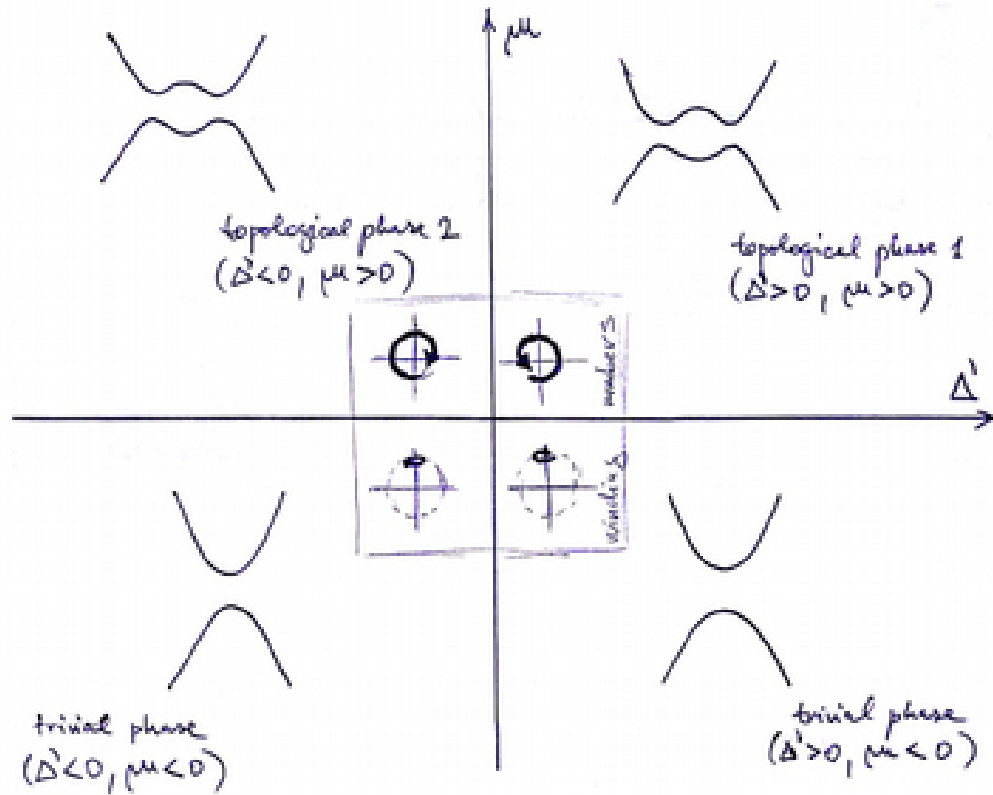
$$H_k = \mathbf{b}_k \cdot \boldsymbol{\tau} \quad \hat{\mathbf{b}}_k = \mathbf{b}_k / \|\mathbf{b}_k\| = \begin{pmatrix} 0 \\ \Delta' k / \sqrt{(\Delta' k)^2 + \xi_k^2} \\ \xi_k / \sqrt{(\Delta' k)^2 + \xi_k^2} \end{pmatrix}$$

Pauli matrices in particle-hole space

Spinless p -wave superconductors

$$E_k = \pm((k^2/2m - \mu)^2 + \Delta'^2 k^2)^{1/2}$$

$$\hat{b}_k = \begin{pmatrix} 0 \\ \Delta' k / \sqrt{(\Delta' k)^2 + \xi_k^2} \\ \xi_k / \sqrt{(\Delta' k)^2 + \xi_k^2} \end{pmatrix}$$



Domain wall Majorana excitations

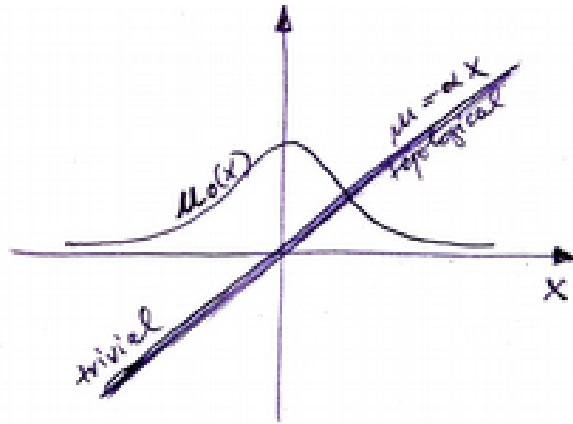
$$\mu(x) = \alpha x$$

$$p^2/2m \ll 1$$

$$\{\tau_i, \tau_j\} = 2\delta_{ij}$$

$$[x, p] = i$$

$$\tau_z \tau_y = -i\tau_x$$

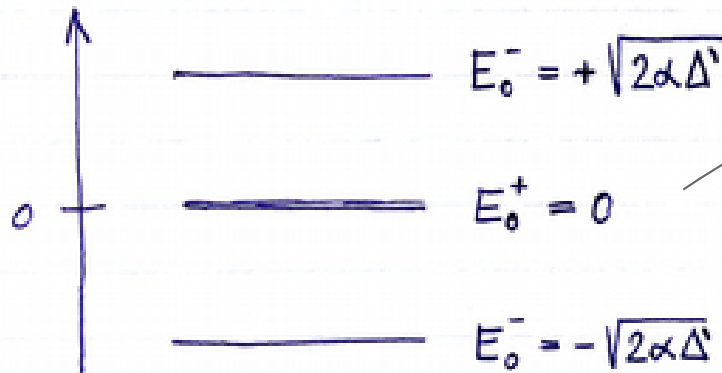


$$H = -\alpha x \tau_z + \Delta' p \tau_y$$

$$H^2 = (\alpha x)^2 + (\Delta' p)^2 - \Delta' \alpha [x, p] \tau_z \tau_y$$

$$H^2 = (\alpha x)^2 + (\Delta' p)^2 - \Delta' \alpha \tau_x$$

$$(E_n^\pm)^2 = 2\Delta' \alpha \left(n + \frac{1}{2}\right) \mp \Delta' \alpha$$



BdG eigenspinor

$$\langle x | n = 0, + \rangle = u_0(x) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\gamma = \int dx u_0(x) [\psi(x) + \psi^\dagger(x)] \longrightarrow \gamma = \gamma^\dagger$$

“Synthetic” realization of Majorana excitations

experimentally accessible system

Basic ingredients:

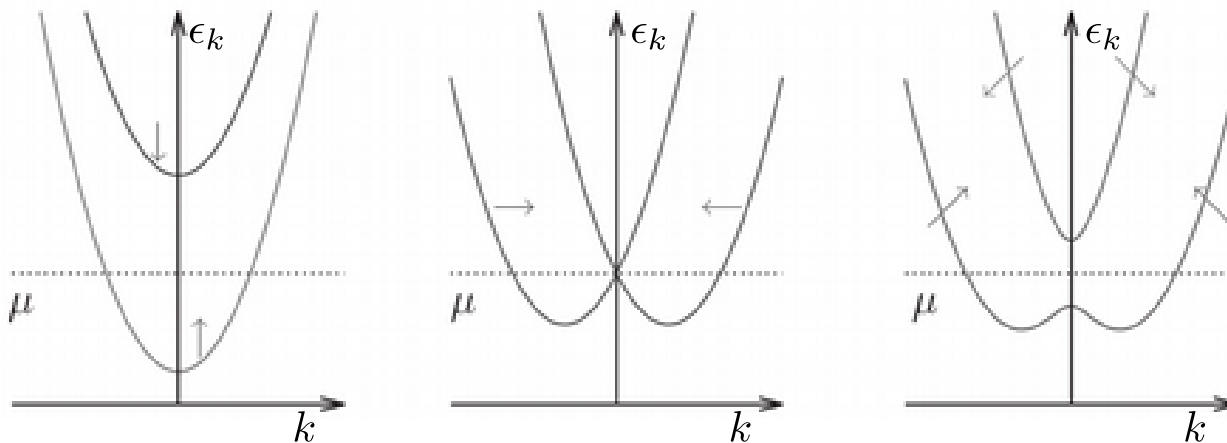
1. proximity coupling to s-wave superconductor
2. spin polarization
3. spin-orbit coupling

$$g\mu_B B > \sqrt{\Delta^2 + \mu^2}$$

$$H = \left(\frac{p^2}{2m} + up\sigma_x - \mu \right) \tau_z - B\sigma_z + \Delta\tau_x$$

Kitev limit (p-wave SC)
strong Zeeman+perturbative SOC

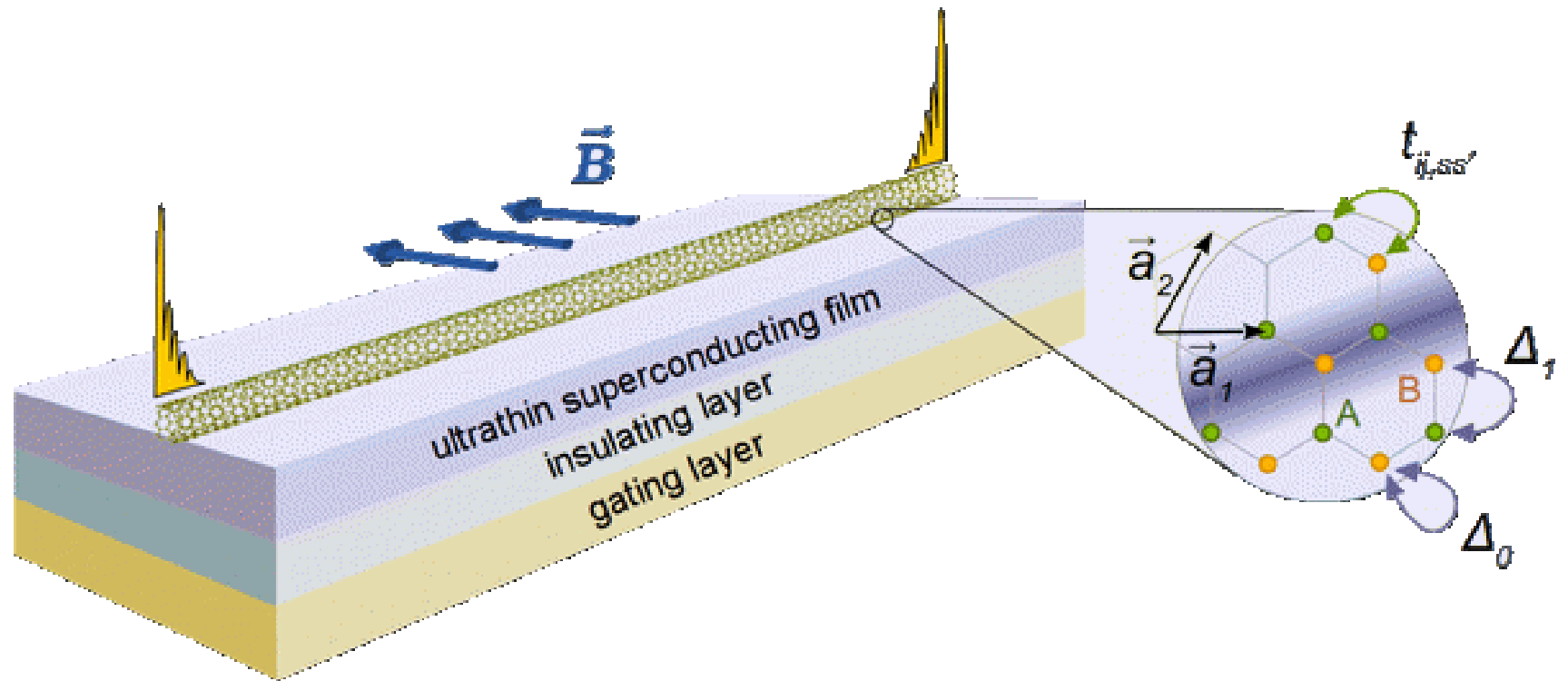
$$H \simeq \left(\frac{p^2}{2m} - \mu \right) \tau_z - \frac{up}{B} \Delta\tau_x$$



effective p-wave pairing strength of the proximity-coupled quantum wire

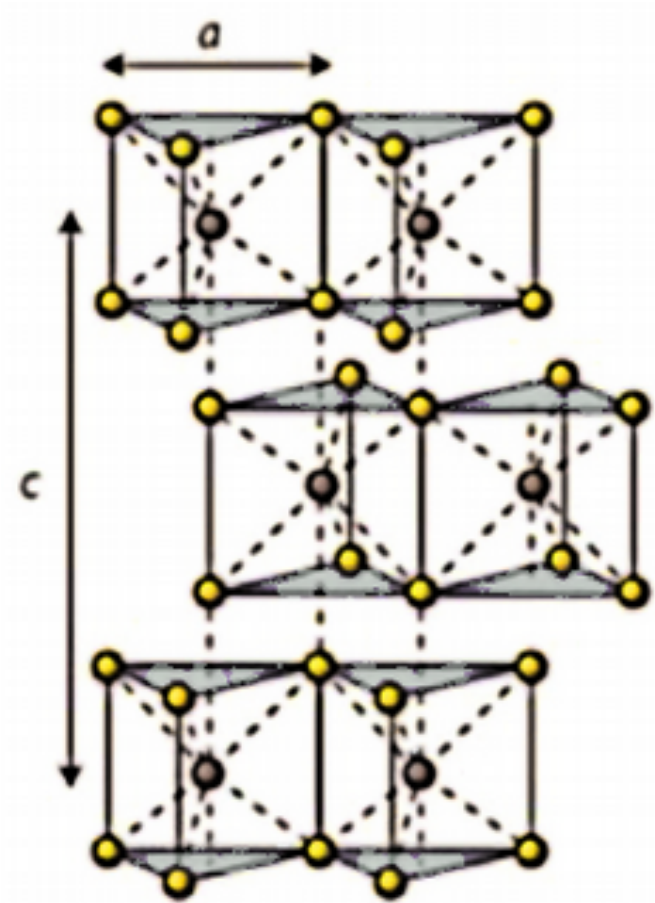
Carbon nanotube hosts Majorana zero mode

novel proposal



[*]M. Marganska et al., arXiv: 1711.03616v1

2H polytype of NbSe₂



SG: #194 ($P6_3/mmc$)

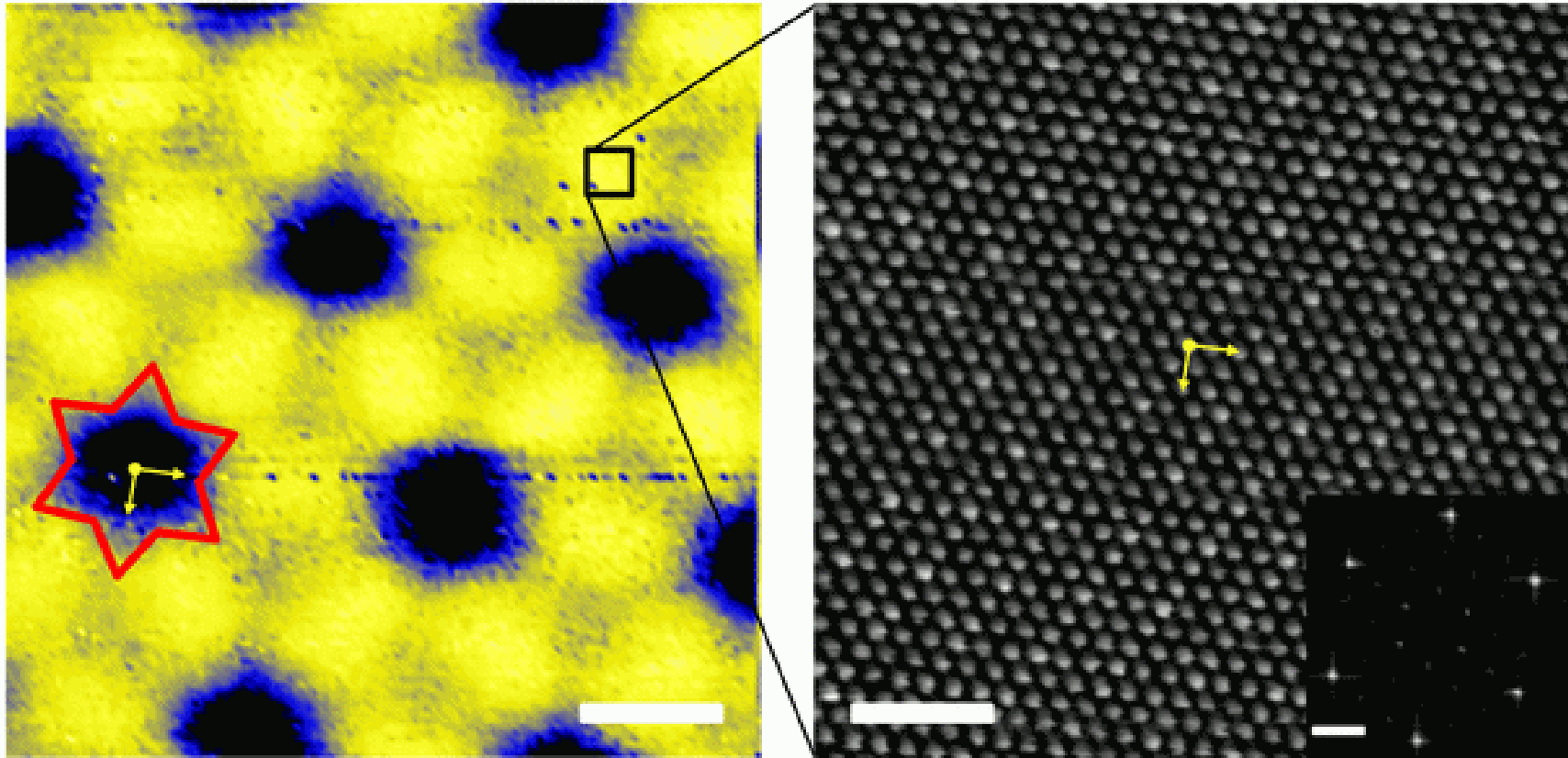
PG: D_{6h}

$a = 3.445 \text{ \AA}$ [*]

$c = 12.55 \text{ \AA}$

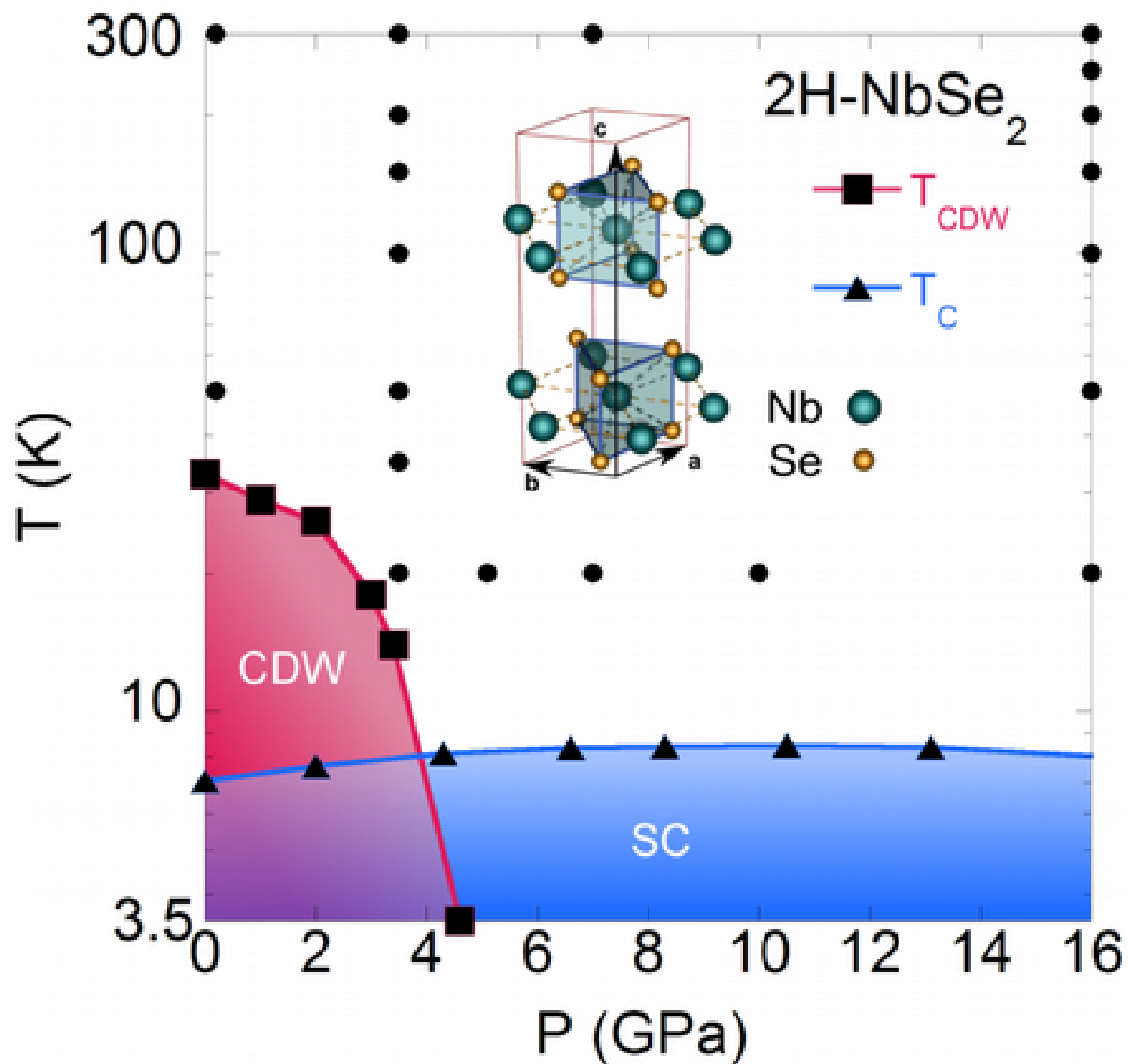
[*]J. Xu et al., Digest Journal of Nanomaterials and Biostructures **10**, 505 (2015)

Anisotropic type-II superconductor



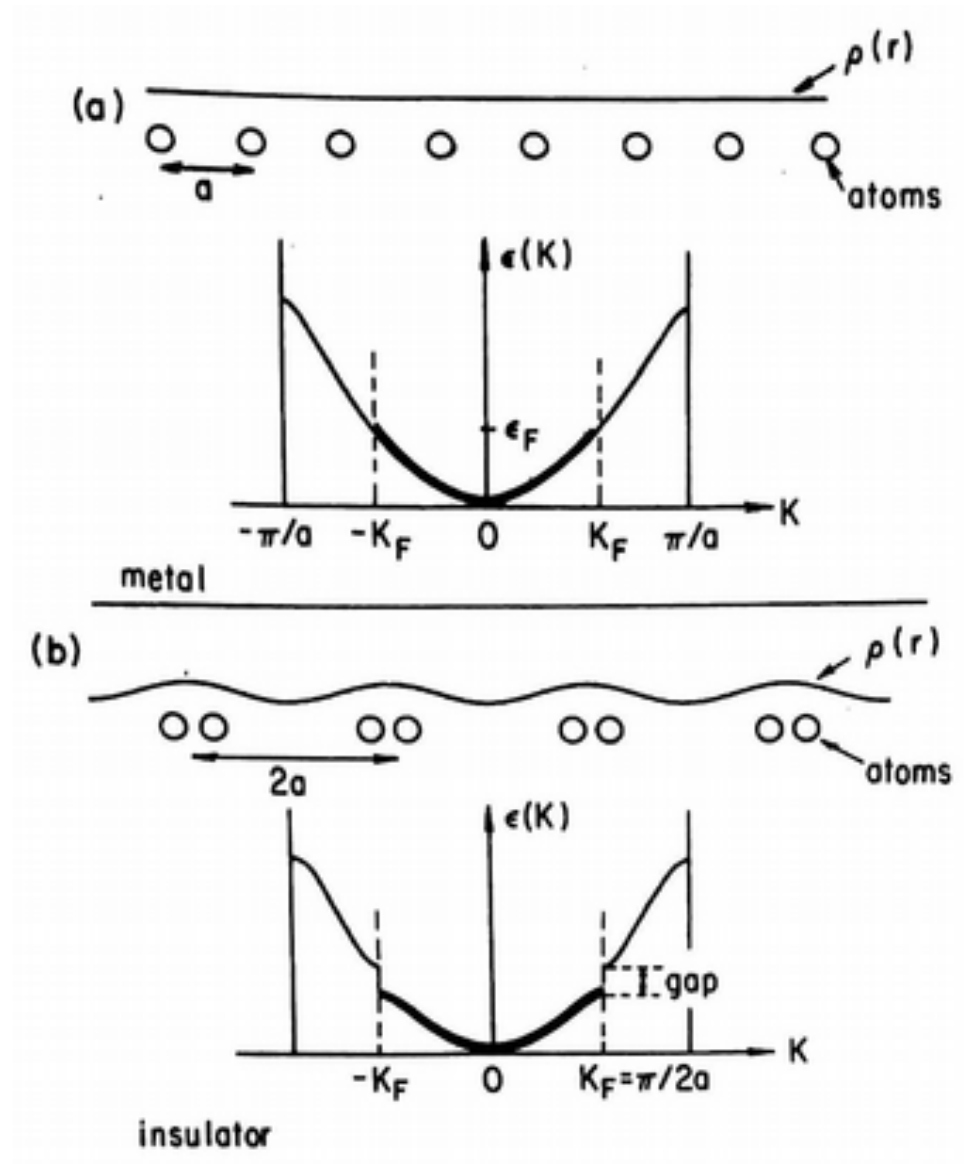
[*]J. A. Galvis et al., arXiv:1711.09269 (2017)

Phase diagram of 2H-NbSe₂



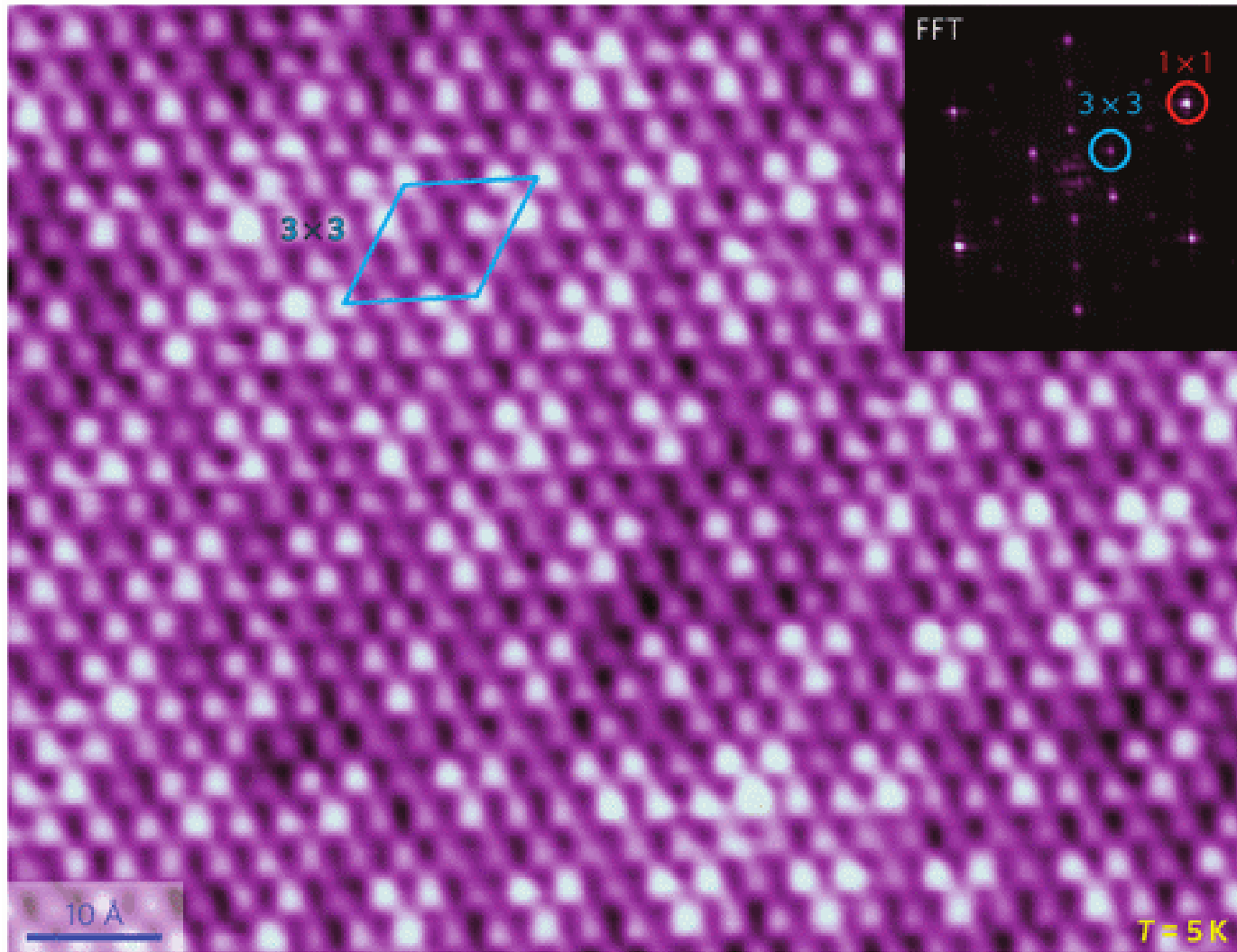
[*]M. Leuroux et al., PRB **92**, 140303 (2015)

Charge density wave (CDW) Peierls transition



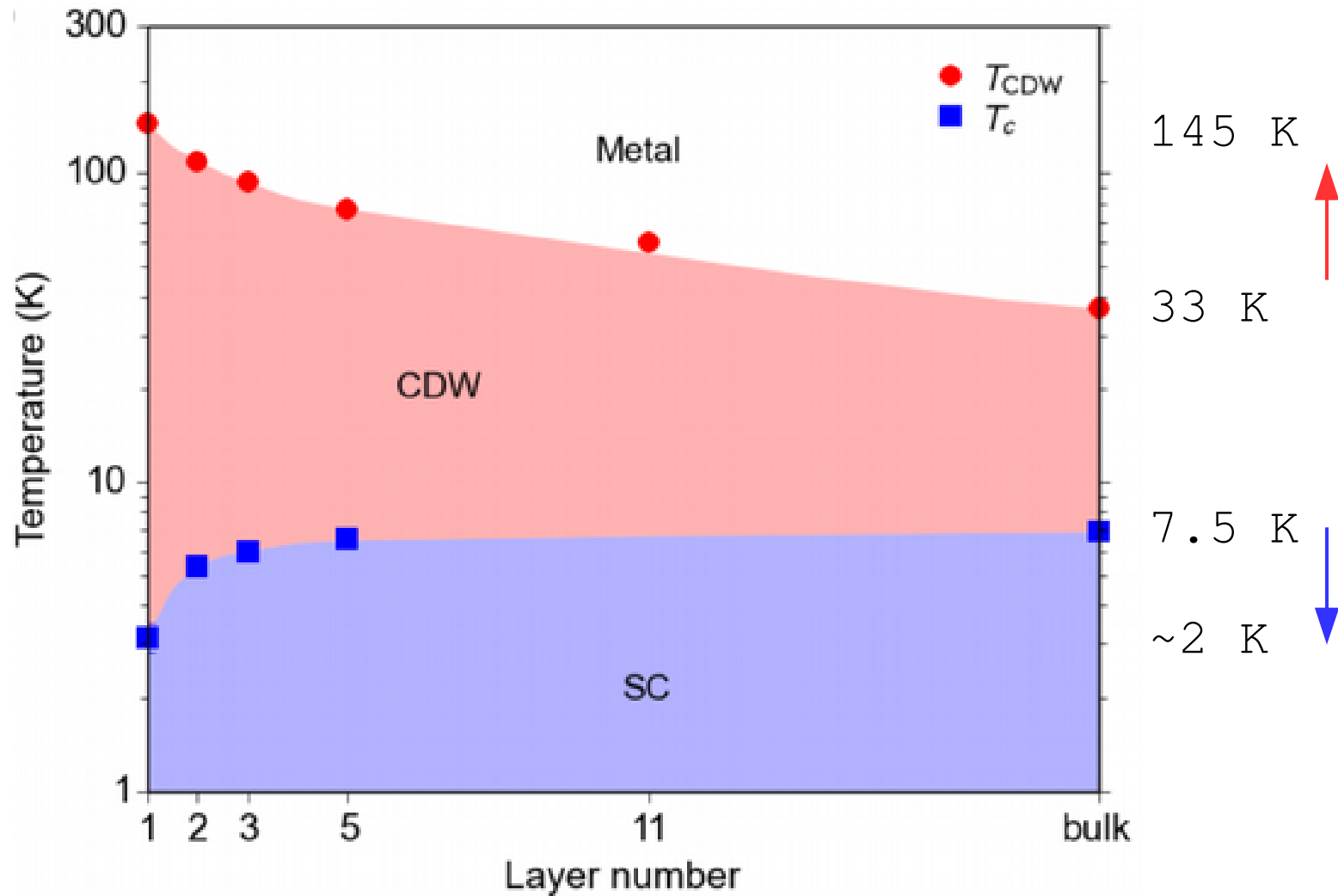
[*]G. Gruener, Rev. Mod. Phys. **60**, 1129 (1988)

3x3 CDW in single layer NbSe₂



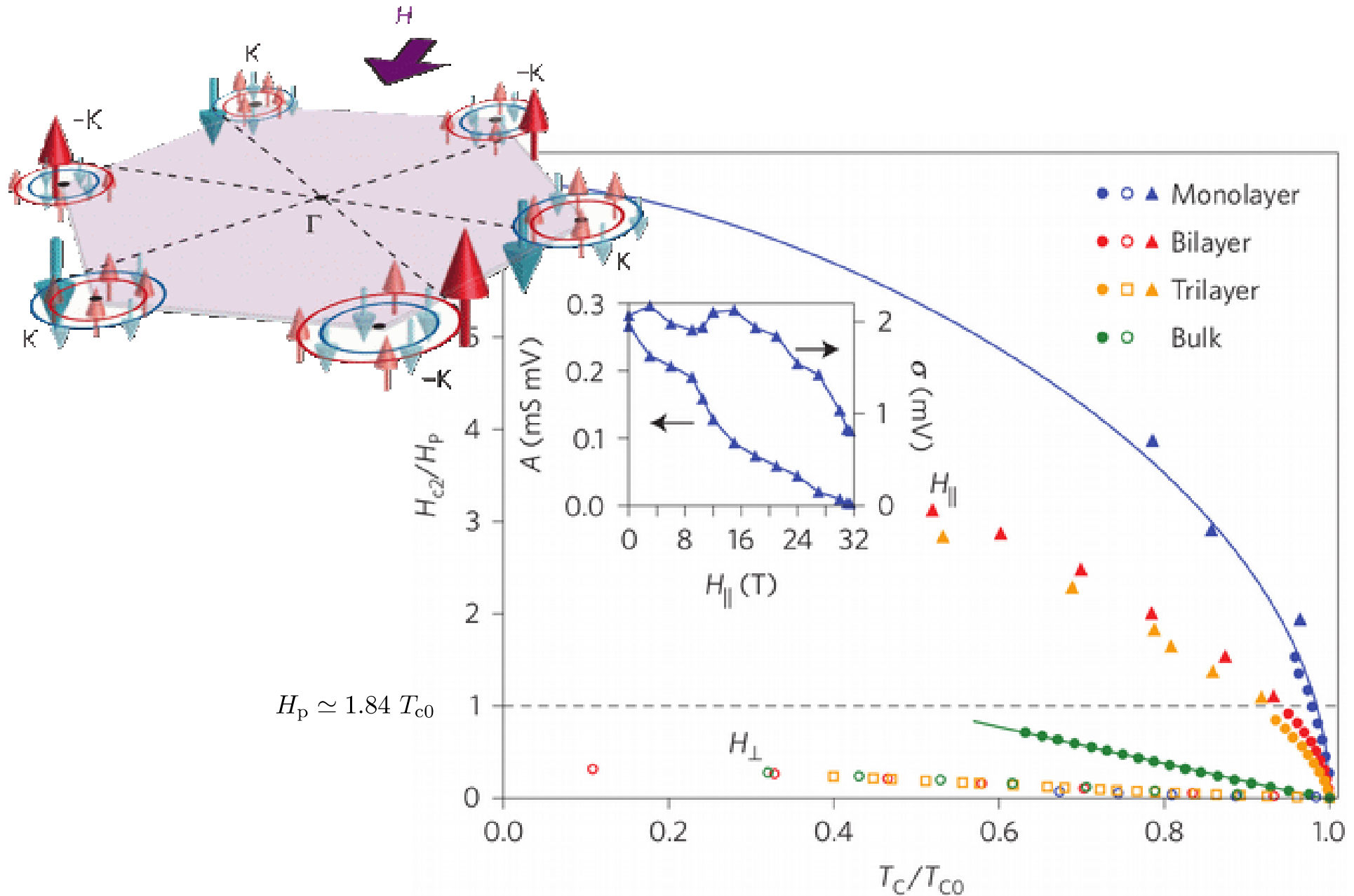
[*]M. M. Ugeda et al., Nat. Phys. **12**, 92 (2016)

Peeling-off NbSe₂ layer-by-layer



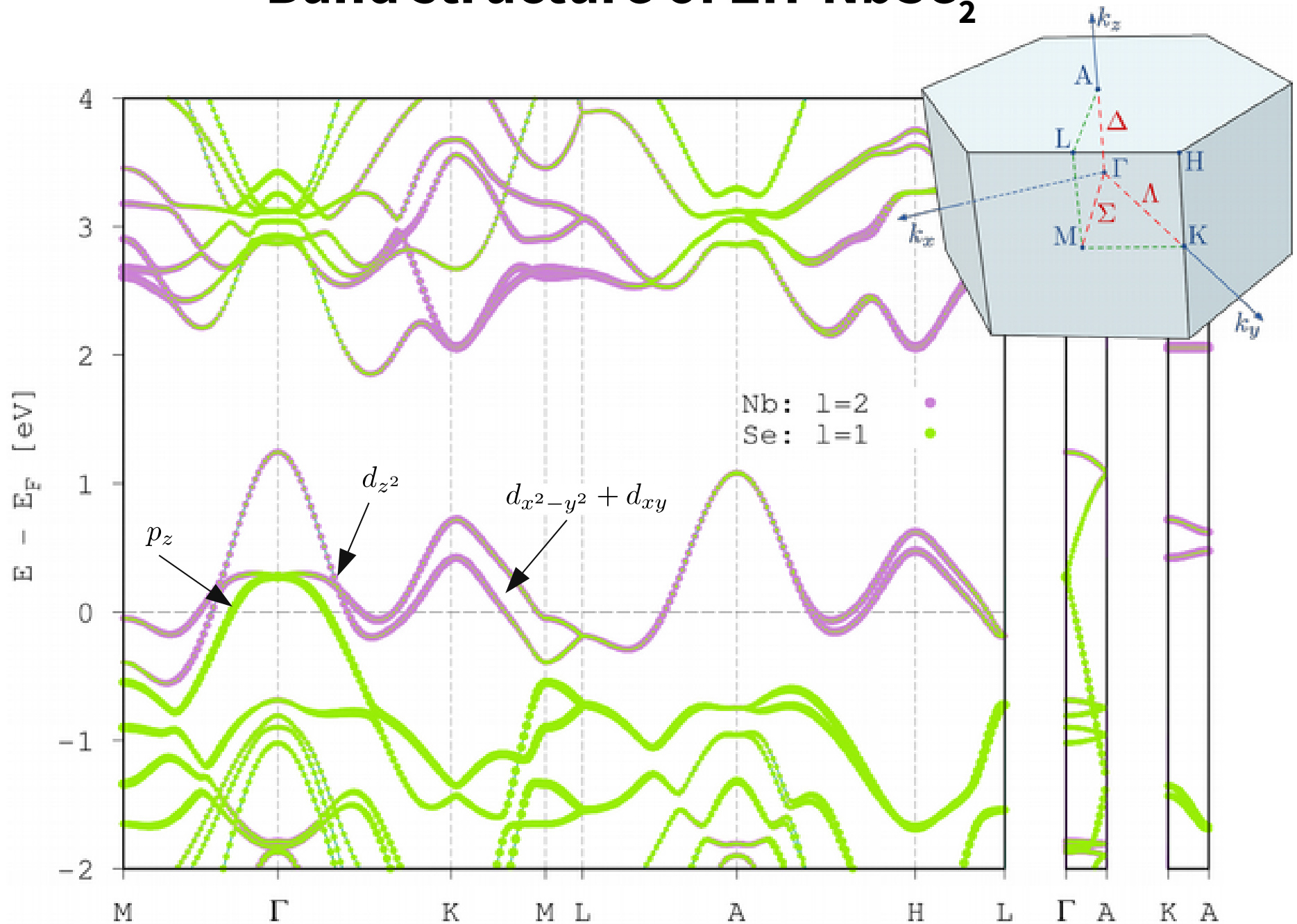
[*]X. Xi et al., Nat. Nanotech. **10**, 765 (2015)

Ising pairing by spin-momentum locking

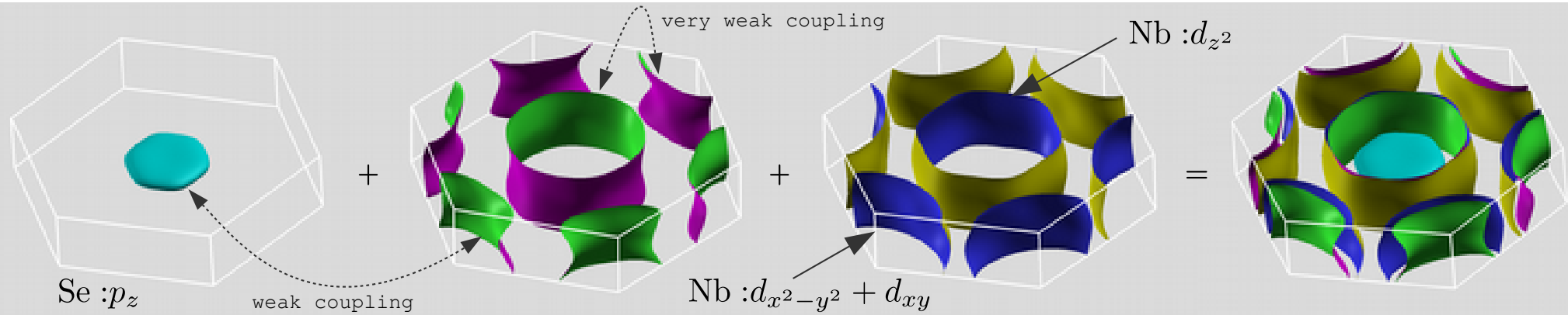


[*]X. Xi et al., Nat. Phys. **12**, 139 (2016); Y. Saito et al., ibid. **12**, 144 (2016)

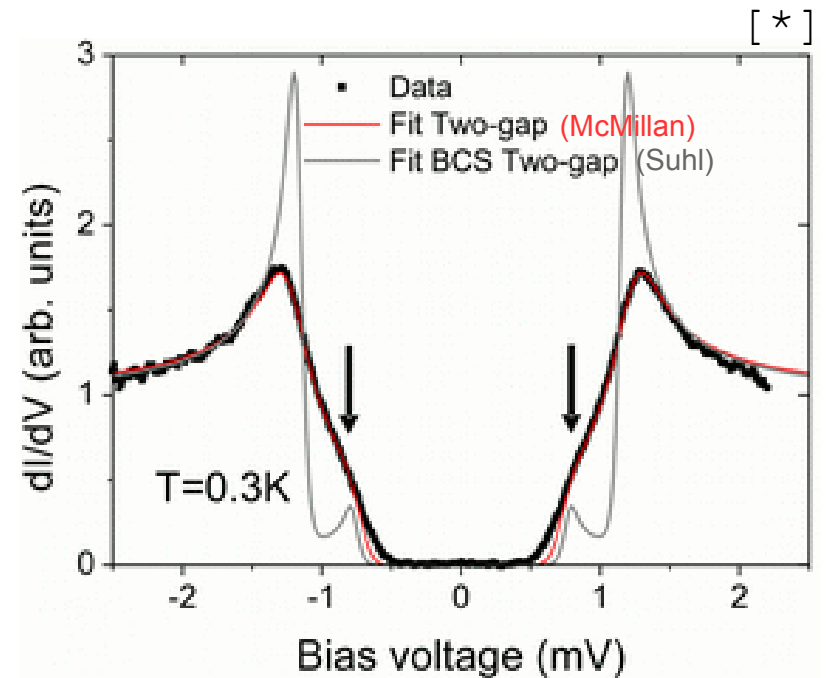
Band structure of 2H-NbSe₂



Fermi surface/superconductivity

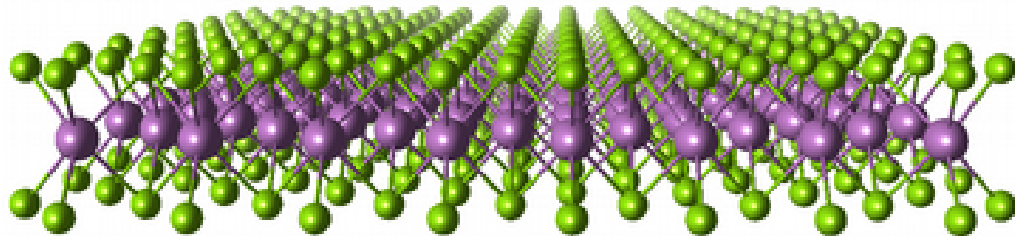
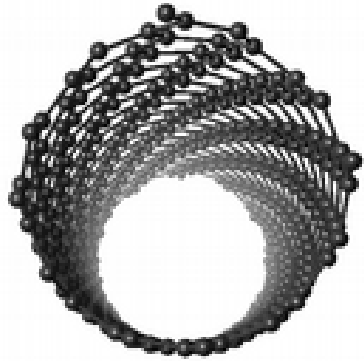


anisotropic gap
or
two-band superconductivity?



[*] Y. Noat et al., PRB **92**, 134510 (2015)

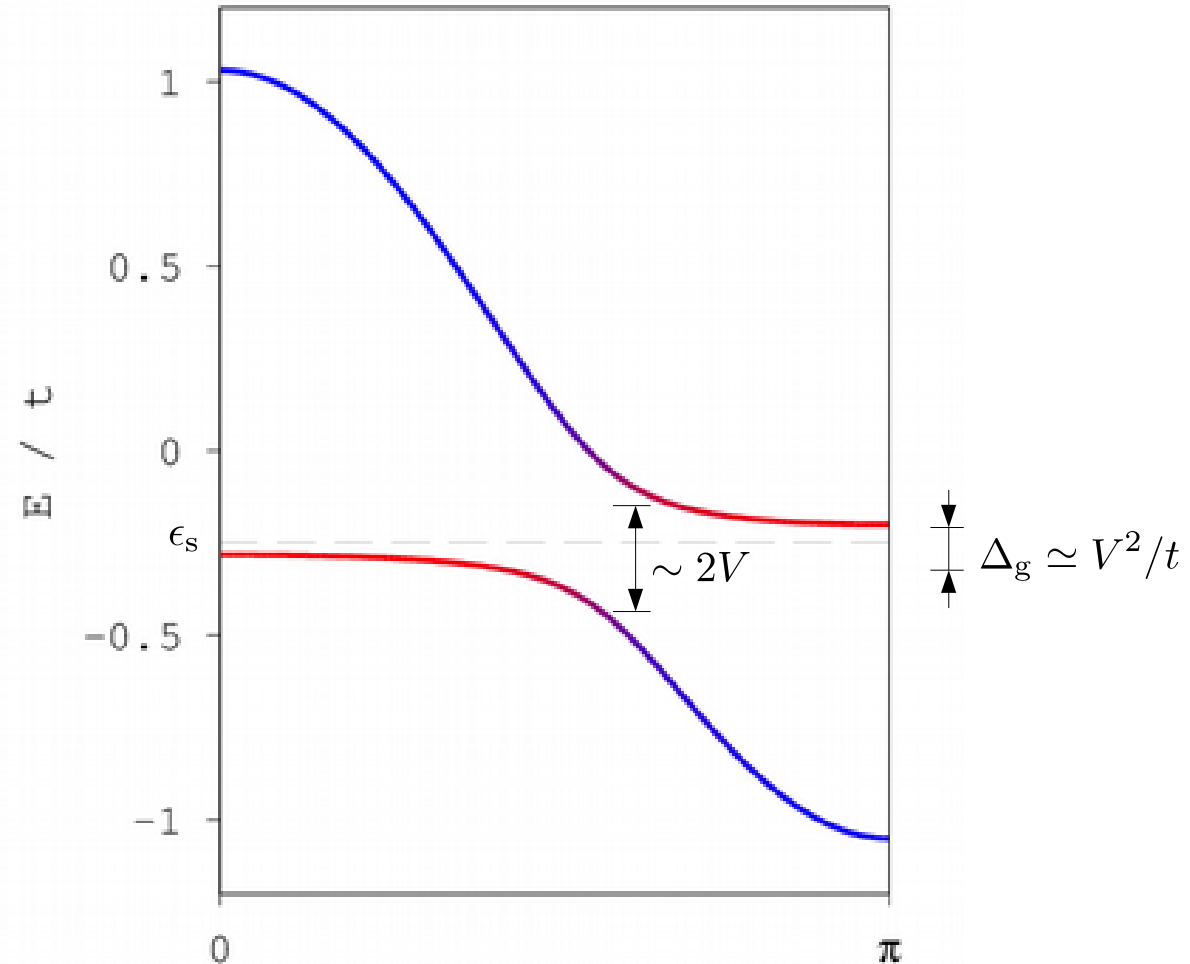
Coupling of two sub-systems



$$\mathcal{H} = \begin{pmatrix} H_0(k) & \Delta(k) \\ \Delta^\dagger(k) & H_s(k) \end{pmatrix}$$

Interaction of two bands

$$\mathcal{H} = \begin{pmatrix} t \cos(k) & V \\ V^* & \epsilon_s \end{pmatrix}$$



[*] solution:
$$E_{1,2} = \frac{1}{2} \left[\epsilon_s + t \cos(k) \pm \sqrt{\epsilon_s^2 + 4V^2 - 2\epsilon_s t \cos(k) + t^2 \cos^2(k)} \right]$$

What would see ARPES?

$$G_{11}^R(\omega, k) = \frac{1}{\omega - t \cos(k) + i\eta - V^2 g_2^R}$$

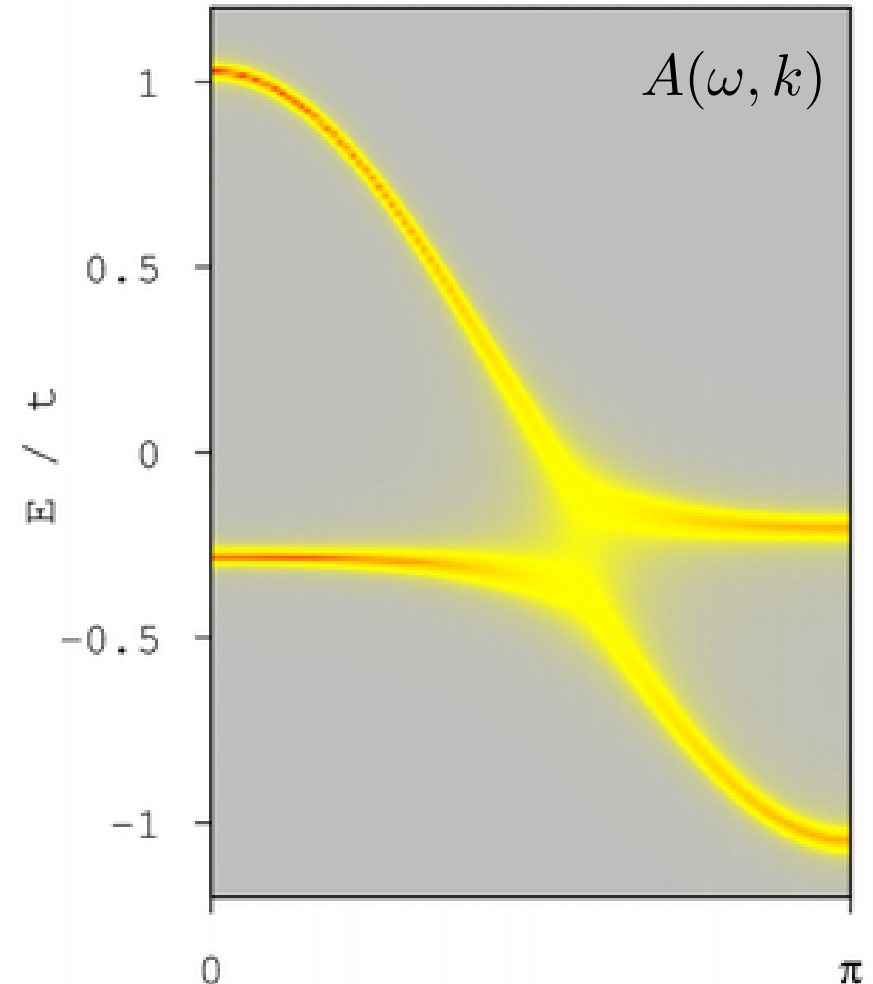
$$G_{22}^R(\omega, k) = \frac{1}{\omega - \epsilon_s + i\eta - V^2 g_1^R}$$

$$\Sigma_{11}(\omega, k) = V^2 g_2^R$$

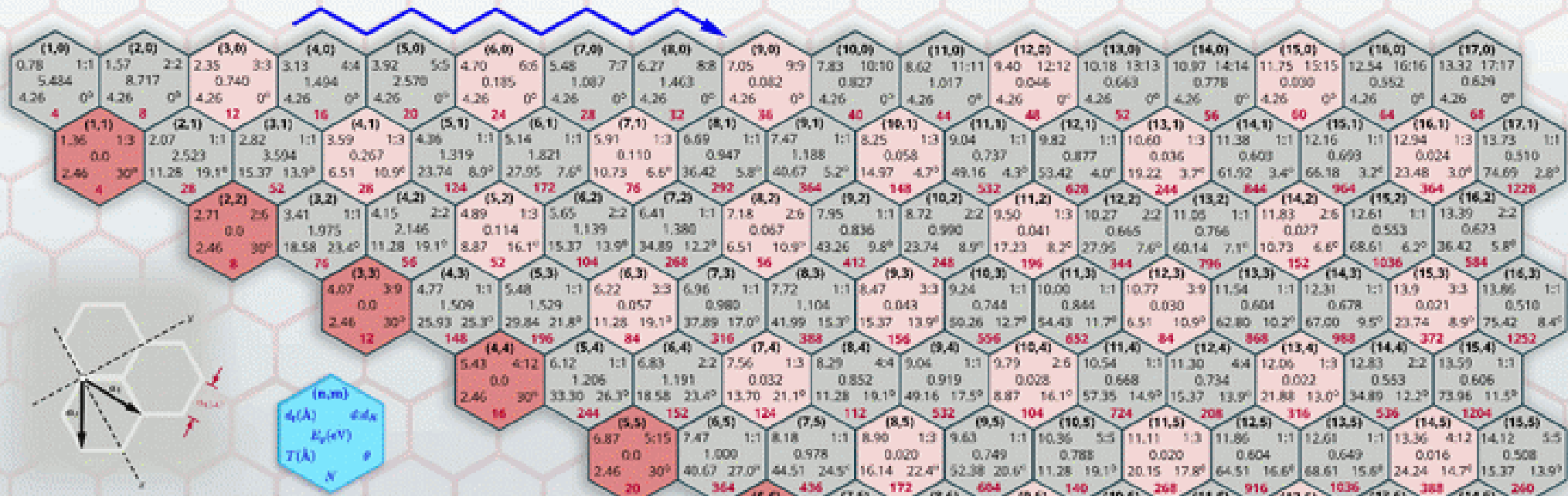
$$\Sigma_{22}(\omega, k) = V^2 g_1^R$$

$$A(\omega, k) = \sum_i A_i(\omega, k)$$


$$A_i(\omega, k) = \frac{-2\text{Im}[\Sigma_{ii}(\omega, k)]}{(\omega - \epsilon_i(k) - \text{Re}[\Sigma_{ii}(\omega, k)])^2 + (\text{Im}[\Sigma_{ii}(\omega, k)])^2}$$



PERIODIC TABLE OF CARBON NANOTUBES



The semi-empirical bandgap E_g is calculated following H. Yonikawa and S. Muramatsu, Phys. Rev. B **52**, 2723 (1995) for the semiconducting tubes (no curvature effects; $|V_{nm}|=2.7$ and $\gamma=0.43$) and A. Kleiner and S. Eggert, Phys. Rev. B **63**, 074028 (2001) for the semi-metallic tubes (includes curvature; $|V_{nm}|=2.7$). All other values are evaluated from the expressions below.



carbon-carbon distance: $a_{CC} = 1.42 \text{ \AA}$ (graphite)

length of unit vector: $a = \sqrt{3}a_{CC} = 2.461 \text{ \AA}$

unit vectors: $a_1 = (\sqrt{3}, 1)/2$ and $a_2 = (\sqrt{3}, -1)/2$

reciprocal unit vectors: $b_1 = (1/\sqrt{3}, 1/2\pi/a)$ and $b_2 = (1/\sqrt{3}, -1/2\pi/a)$

chiral vector: $C_n = na_1 + ma_2$ (where $n, m \in \mathbb{Z}$)

circumference of tube: $L = |C_n| = a\sqrt{n^2 + m^2 + nm}$ (where $0 \leq m \leq n$)

diameter of tube: $d = L/\pi$

chiral angle: $\theta = \angle(a_1, C_n) = \arctan\left(\frac{m\sqrt{3}}{2n-m}\right) \in [0^\circ, 30^\circ]$

lowest common divisor of n, m : $d = \text{lcd}(n, m)$

lowest common divisor of $2n+1, m, 2m+n$: $d_g = \text{lcd}(2n+1, 2m+n)$
 $\begin{cases} d, & \text{if } n-m \text{ is not a multiple of } 3d \\ 3d, & \text{if } n-m \text{ is a multiple of } 3d \end{cases}$

translational vector of 1D unit cell: $T = l_1 a_1 + l_2 a_2$ (where $l_1, l_2 \in \mathbb{Z}$)
 $l_1 = (2n+1)/d_g$
 $l_2 = (2m+n)/d_g$

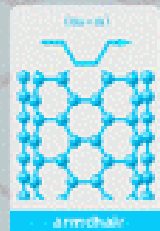
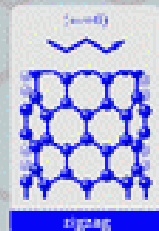
length of T: $|T| = L\sqrt{3}/d_g$

number of atoms per 1D unit cell: $N = (2\pi)^2/(a^2 d_g)$, and $N/2$ helicity/spin/unit cell

metallic tube

semimetallic tube

semiconducting tube

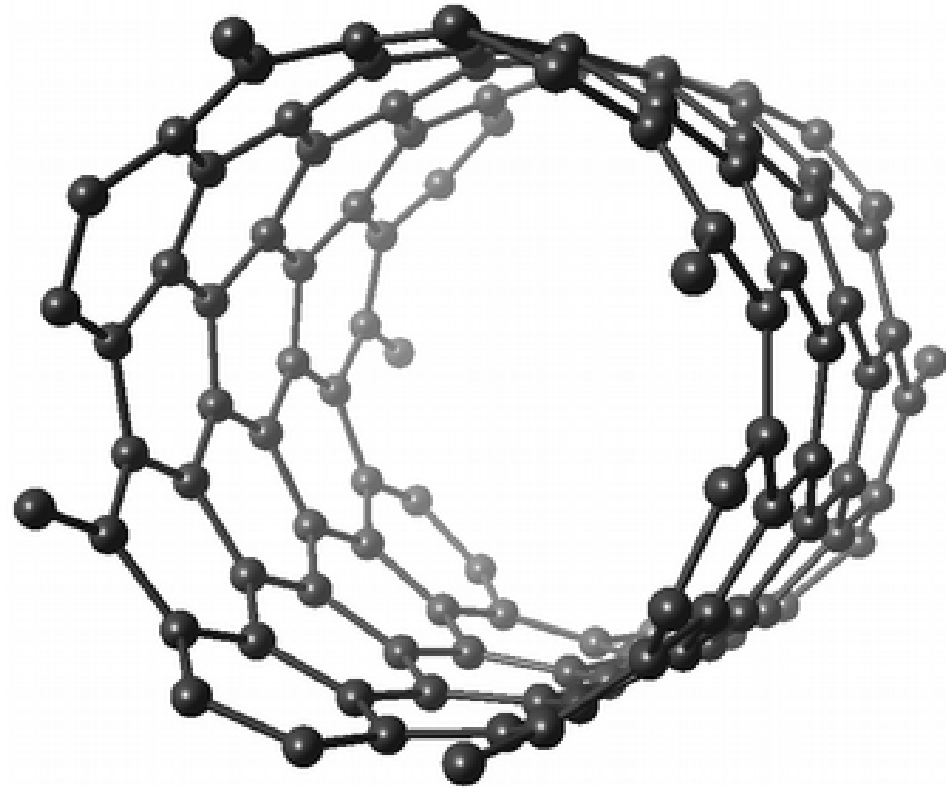
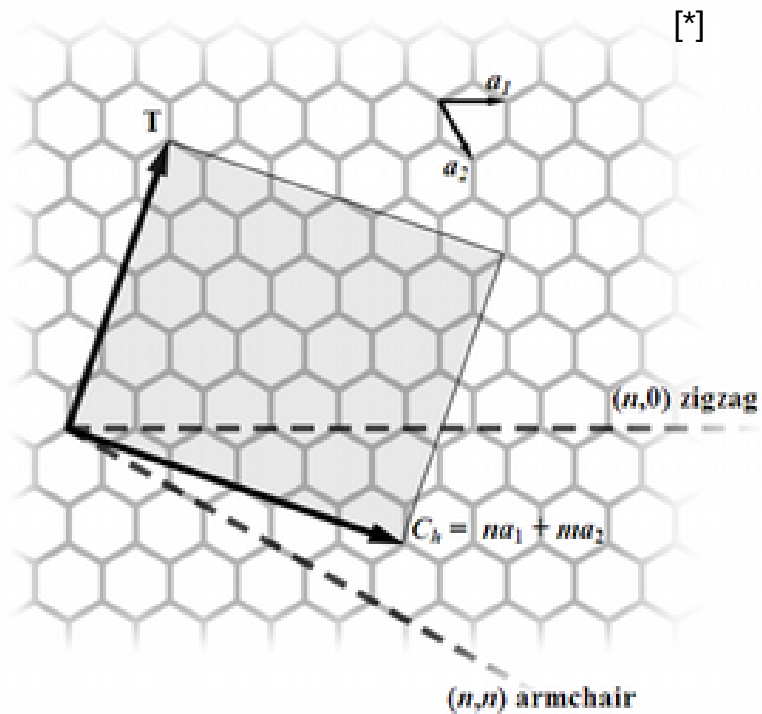


Quantum Wise

When every atom matters



Chiral carbon (8,4) nanotube

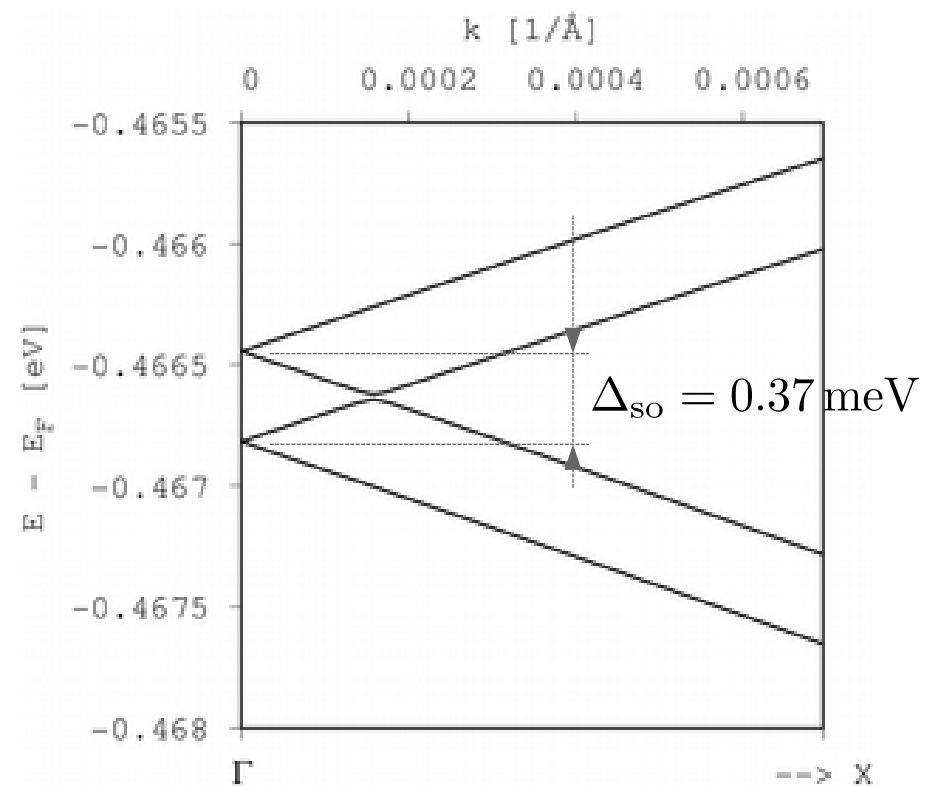
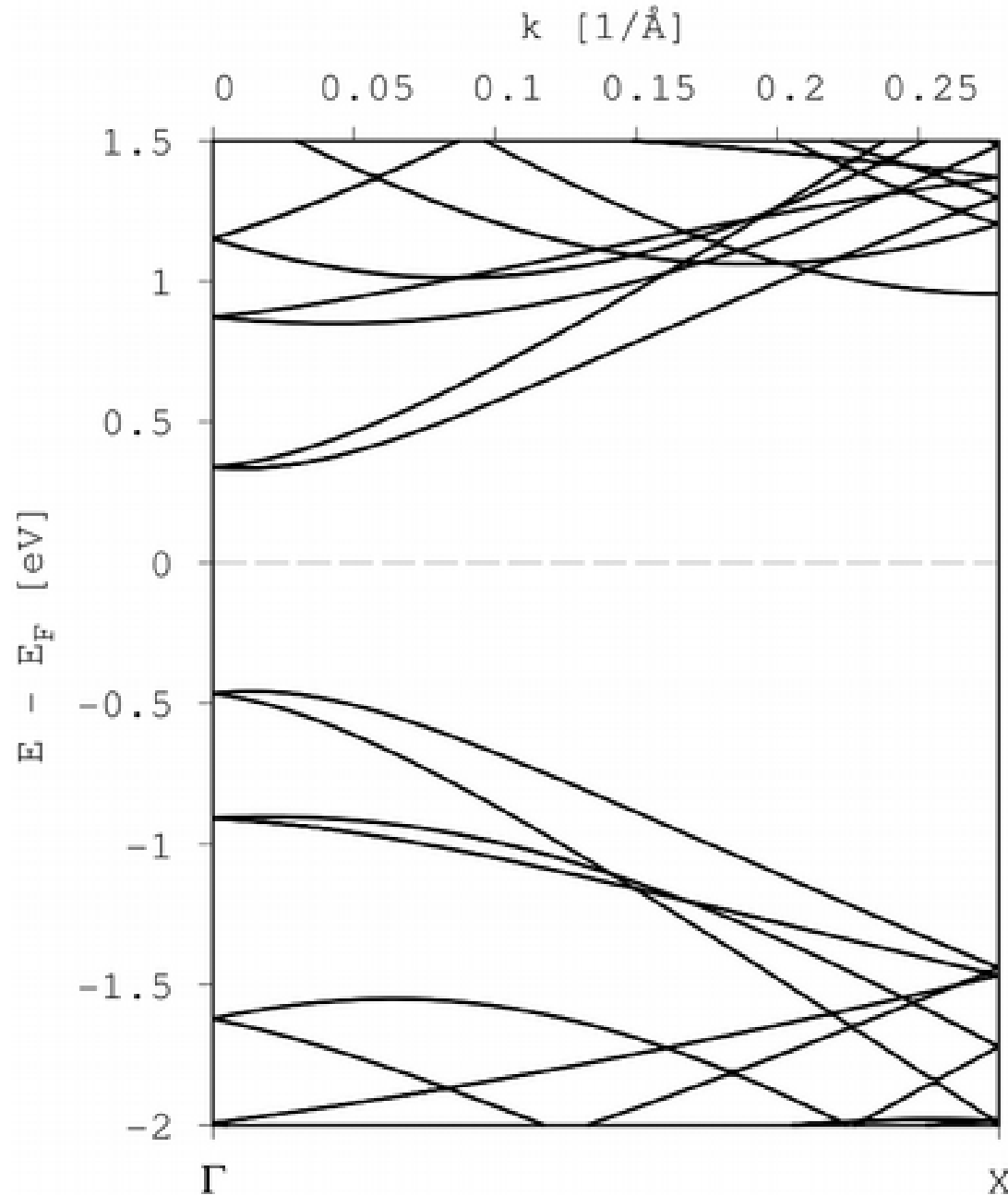
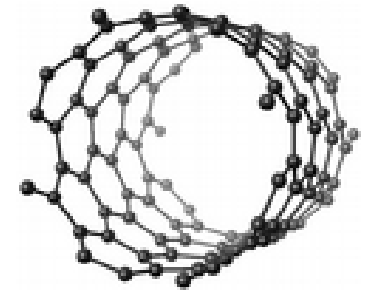


diameter of 8.3 Å
112 atoms in unit cell^[**]

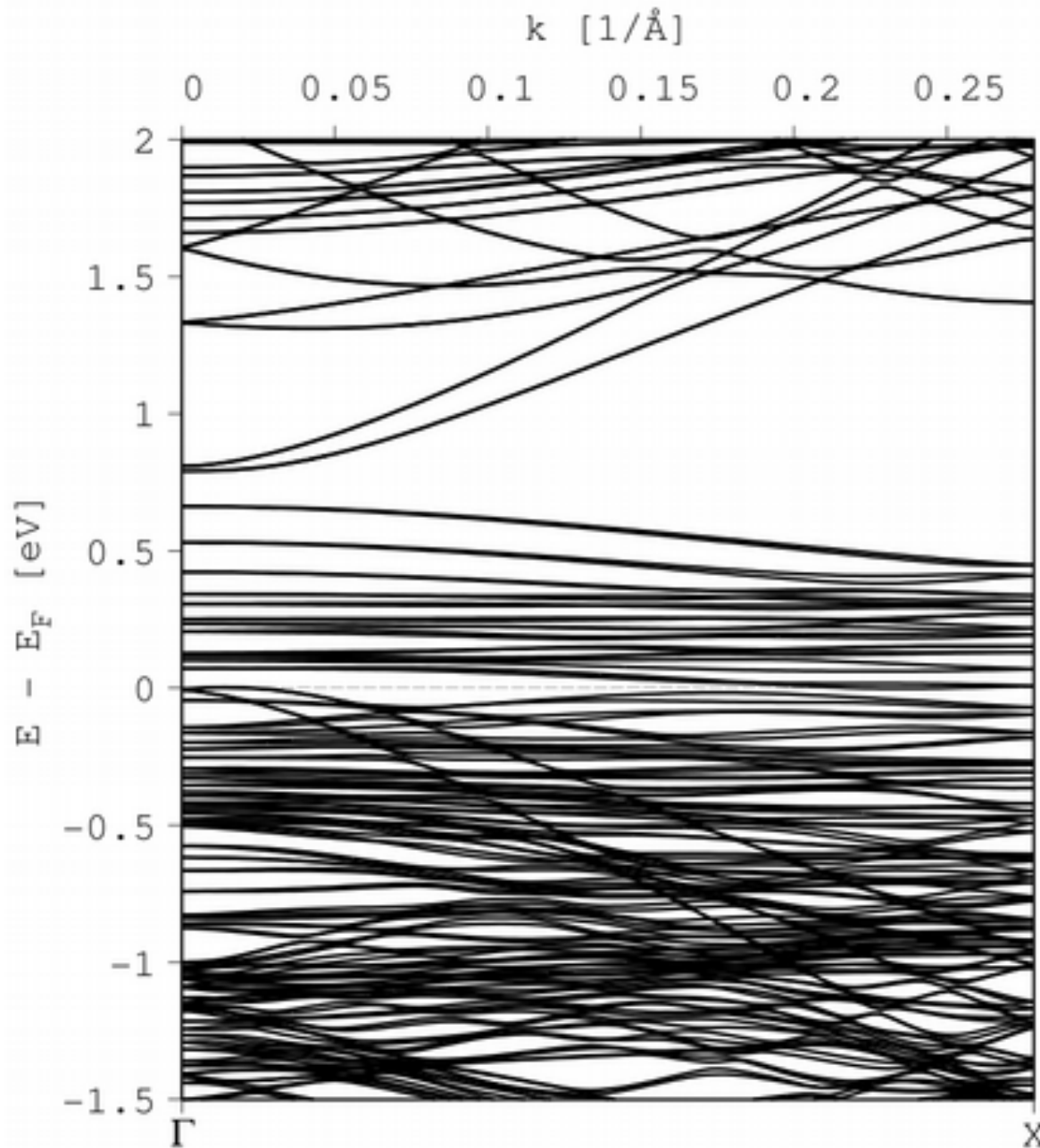
[*]source: https://en.wikipedia.org/wiki/Carbon_nanotube

[**]input: TubeGen3.3, J T Frey, University of Delaware

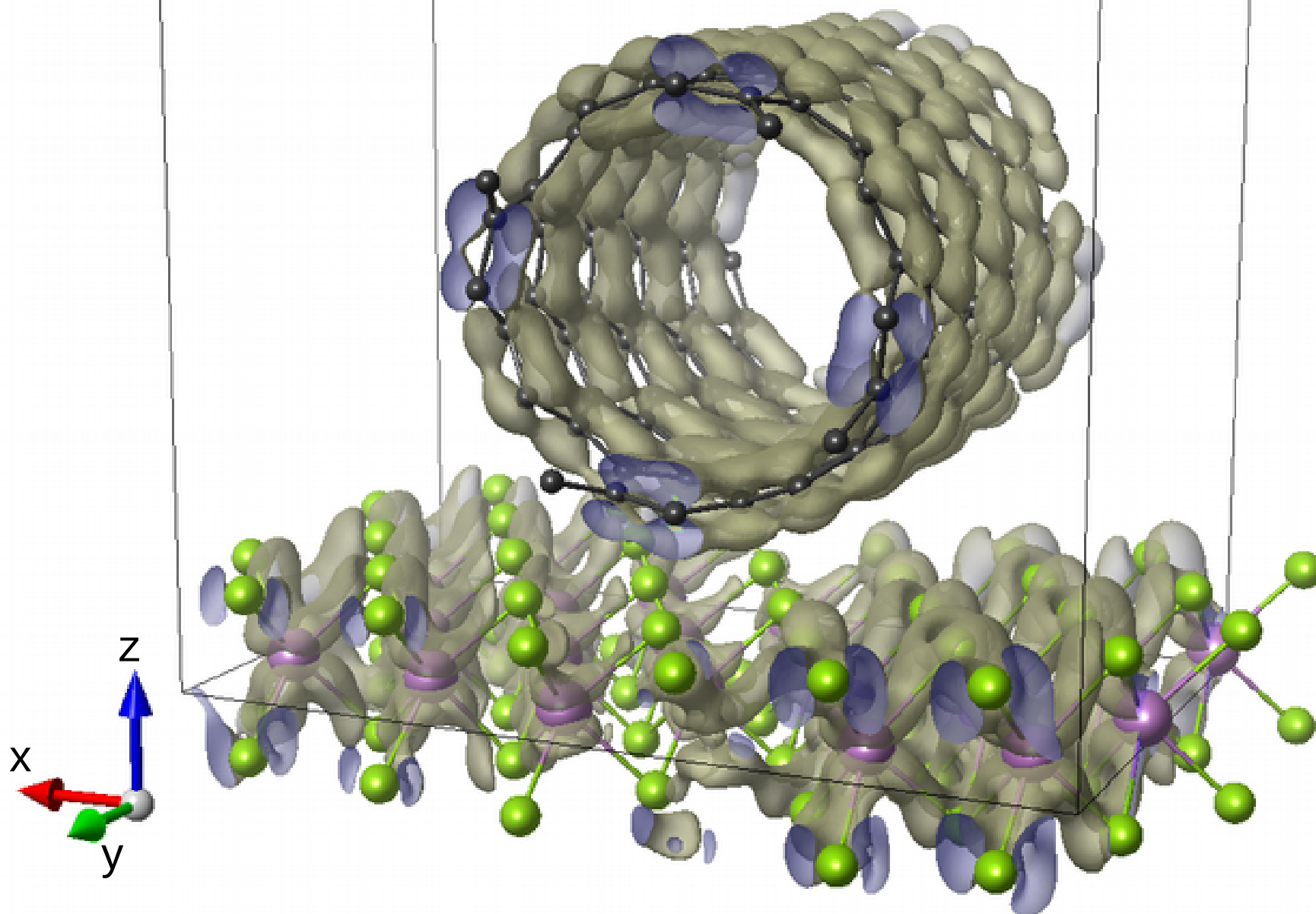
Band structure of cnt(8,4)



Joint spaghetti

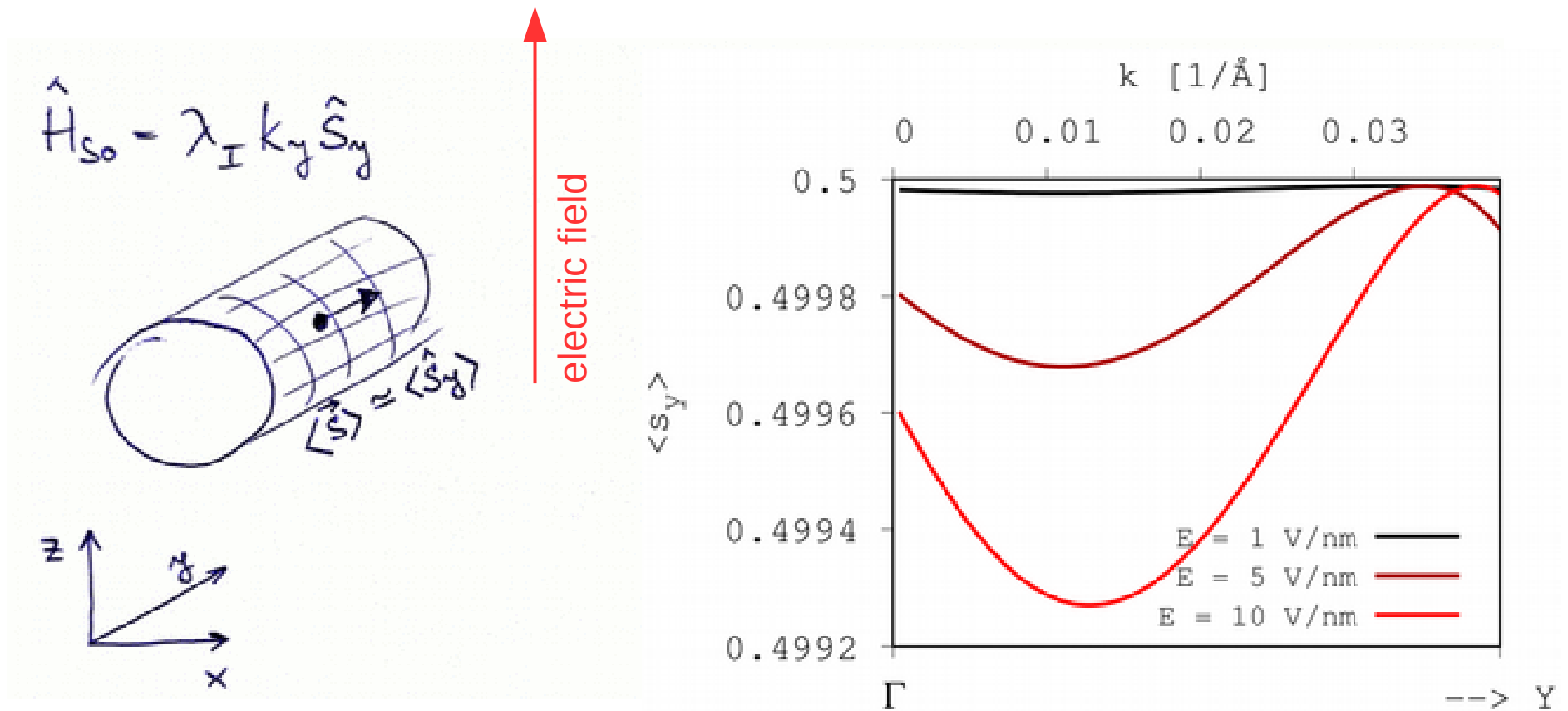


Hybridization state



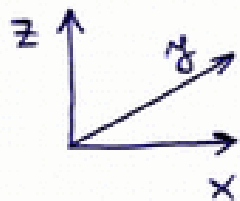
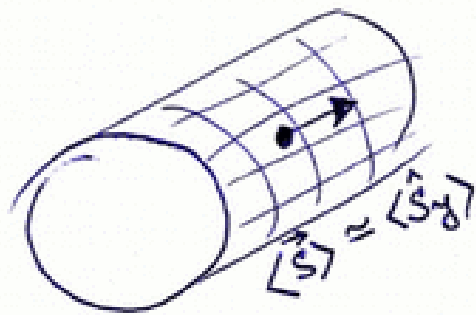
$[\ast]k=0.0216 \text{ 1/\AA}$ (path G-Y, K#9, band 1046)

Spin-orbit coupling form

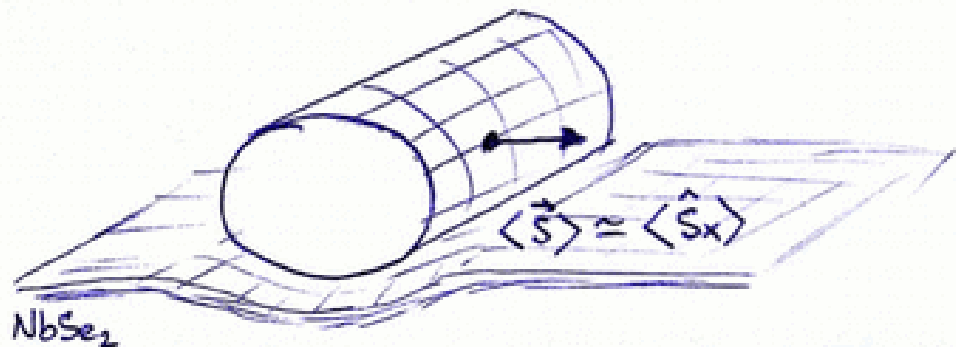


Spin-orbit coupling form

$$\hat{H}_{\text{so}} = \lambda_{\text{I}} k_y \hat{S}_y$$

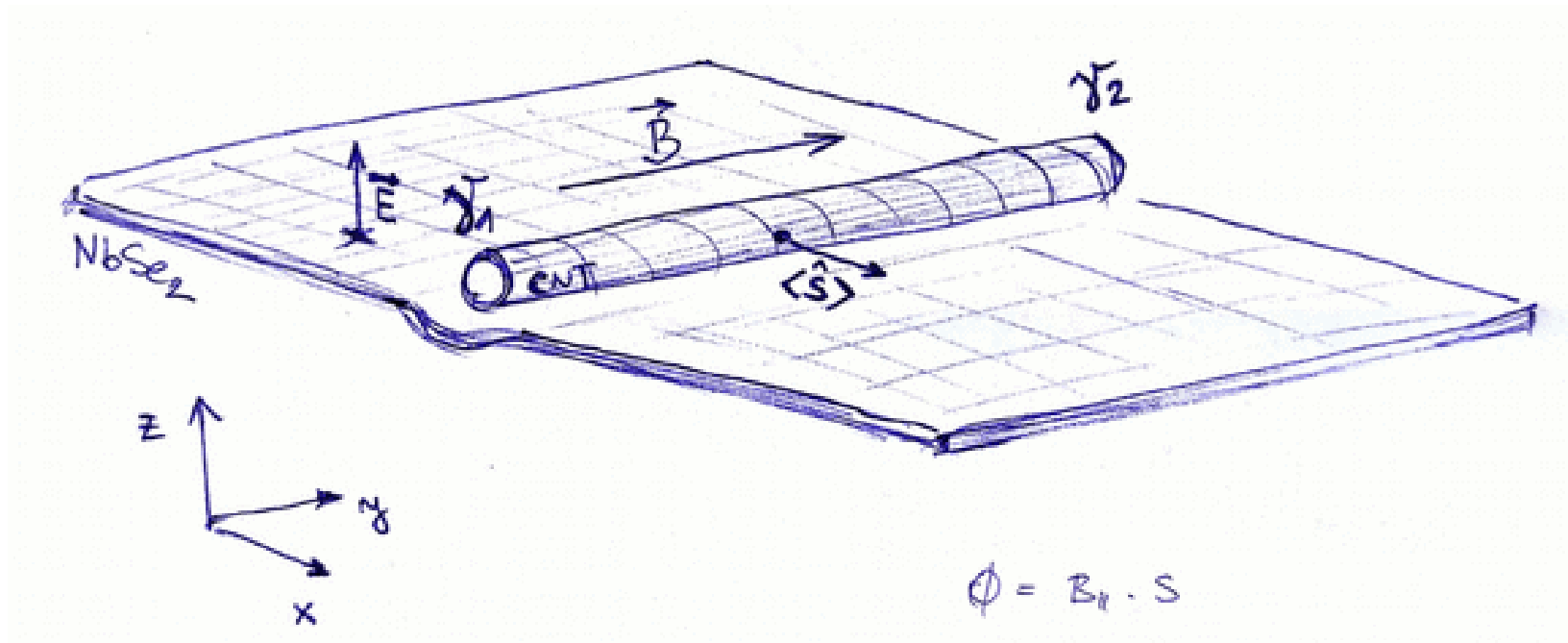


$$\hat{H}_{\text{so}} = \lambda_{\text{I}} k_y \hat{S}_y + \lambda_{\text{E}} k_y \hat{S}_x$$



$$\lambda_{\text{I}} \ll \lambda_{\text{E}}$$

Save the Majoranas!



$$\Phi = B_{\parallel} \cdot S$$

$$B_{\parallel} = 10 \text{ T}$$

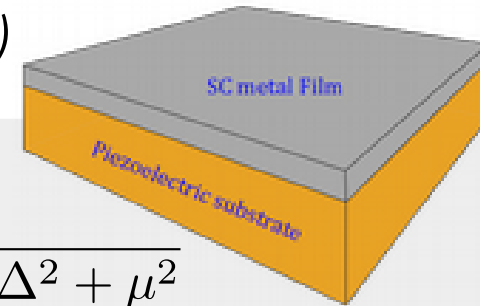
$$S = \pi R^2 = \pi \left(\frac{8.3 \text{ \AA}}{2} \right)^2 = 54.1 \text{ \AA}^2$$

$$\Phi = 1.308 \times 10^{-5} \phi_0$$

[*]Note: magnetic splitting in 10 Tesla would be 1 meV

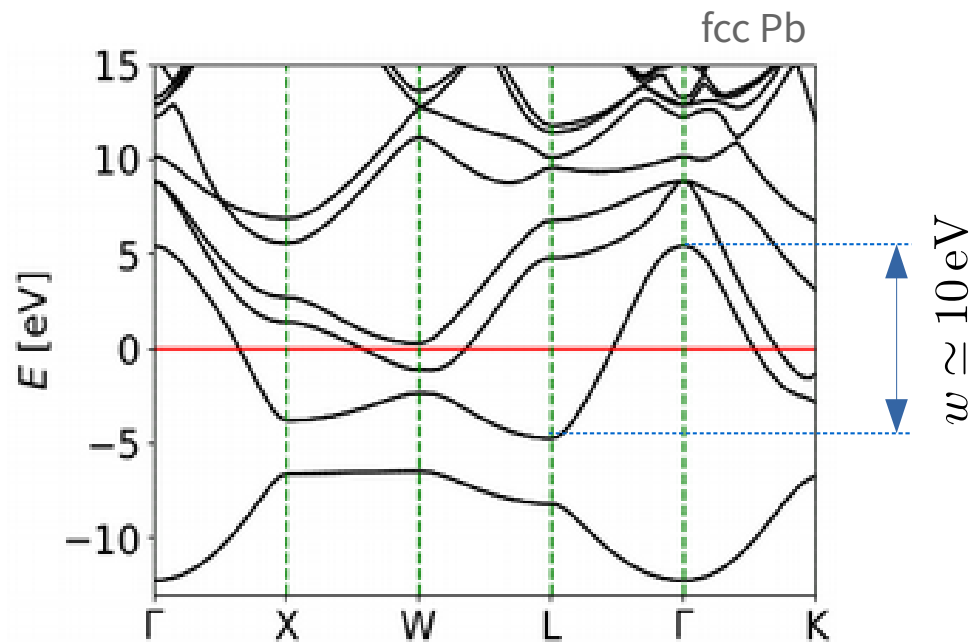
Ultrathin films of superconductors

a platform for topological superconductivity (Pb or β -Sn)



- Cooper pairing
- broken inversion symmetry
- broken time-reversal symmetry

$$\Delta_z = g\mu_B B > \sqrt{\Delta^2 + \mu^2}$$



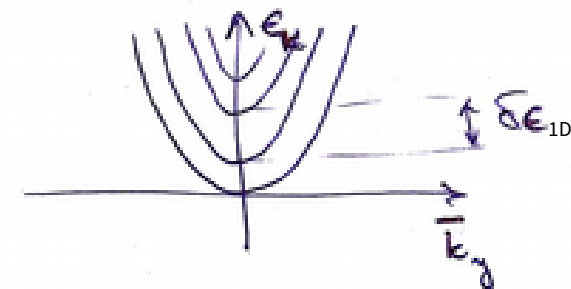
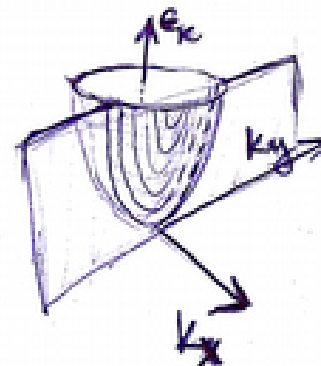
large g -factor $\sim 100 - 350$
 E-field 0.1 V/nm ~ 2 meV subbands shift
 strain of 1% ~ 200 meV Fermi level tuning

$$w/2N \simeq \delta\epsilon_{2D}$$

$$H_{c,\perp} \approx 1.6 \text{ T}$$

$$H_{c,\parallel} \approx 55 \text{ T}$$

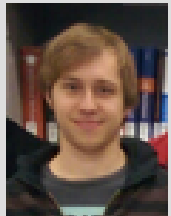
Advantage: interface or proximity effect free system to obtain SC in a strong SOC systems



Conclusions

- **symmetry dictates topology** of spin-orbit coupling fields
- **appreciable proximity** induced spin-orbit coupling effects in vdW heterostructures
- **carbon nanotubes** could host **Majorana** bound states
- promising topological superconductivity in **thin films of heavy elements**

Acknowledgments



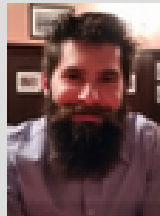
Tobias Frank



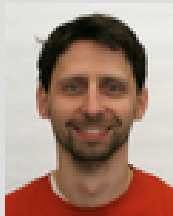
Klaus Zollner



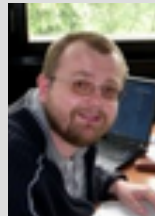
Petra Högl



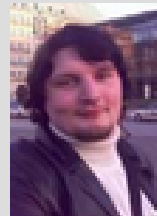
Paulo E. Faria



Marcin Kurpas



Denis Kochan



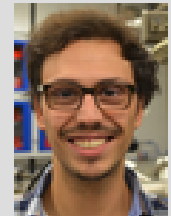
Sergej Konschuh



Alex Matos



Magda Marganska



Flo Dirnberger



Jaroslav Fabian